

1.4 Converting From Decimal

$$\frac{19}{8} = 2 + \frac{3}{8}$$

2 is the quotient (Q for short)

3 " remainder (R for short)

$$19 \div 8 = 2.375$$

$$\begin{array}{r} Q=2 \\ \hline R \end{array}$$

$$R = 8 \times 0.375 = 3$$

Ex: Find Q and R

$$\begin{aligned} 133 \div 16 \\ = 8.3125 \end{aligned}$$

$$Q=8$$

$$R = 16 \times 0.3125 = 5$$

To convert from decimal to another base
we do repeated division.

Ex: Convert 106_{10} to octal (base 8)

$$\begin{array}{r} & Q & R \\ 106 \div 8 & \boxed{13} & 2 \\ 13 \div 8 & | & 5 \\ 1 \div 8 & 0 & | \end{array}$$

Stop when you reach $Q=0$.

Answer is R-values, in reverse order.

$$106_{10} = 152_8$$

Ex: Convert 58 to binary (base 2).

| | Q | R |
|-------------|----|---|
| $58 \div 2$ | 29 | 0 |
| $29 \div 2$ | 14 | 1 |
| $14 \div 2$ | 7 | 0 |
| $7 \div 2$ | 3 | 1 |
| $3 \div 2$ | 1 | 1 |
| $1 \div 2$ | 0 | 1 |

$$58 = 111010_2$$

Ex: Convert 1792 to octal (base 8)

| | Q | R |
|---------------|-----|---|
| $1792 \div 8$ | 224 | 0 |
| $224 \div 8$ | 28 | 0 |
| $28 \div 8$ | 3 | 4 |
| $3 \div 8$ | 0 | 3 |

$$1792 = 3400_8$$

Ex: Convert 547 to base 5

| | Q | R |
|--------------|-----|---|
| $547 \div 5$ | 109 | 2 |
| $109 \div 5$ | 21 | 4 |

$$\begin{array}{r}
 21 \div 5 \\
 4 \div 5
 \end{array}
 \quad
 \begin{array}{c|cc}
 & 4 & 1 \\
 & 0 & 4
 \end{array}
 \quad
 \begin{array}{c}
 \uparrow \\
 \uparrow
 \end{array}$$

$$547 = 4142_5$$

Ex: Convert 537_{10} to hexadecimal
(base 16)

$$\begin{array}{r}
 537_{10} \div 16 \\
 3356 \div 16 \\
 209 \div 16 \\
 13 \div 16
 \end{array}
 \quad
 \begin{array}{c|cc}
 Q & R \\
 \hline
 3356 & \cancel{14E} \\
 209 & \cancel{12C} \\
 13 & 1 \\
 0 & \cancel{13D}
 \end{array}
 \quad
 \begin{array}{l}
 \uparrow \\
 \uparrow
 \end{array}$$

| |
|---------------|
| $A_{16} = 10$ |
| $B_{16} = 11$ |
| $C_{16} = 12$ |
| $D_{16} = 13$ |
| $E_{16} = 14$ |

$$537_{10} = D1CE_{16}$$

To convert non-integer numbers from decimal to another base, we do repeated multiplication.

Ex: Convert 0.375 to binary.

$$\begin{array}{r}
 0.375 \times 2 \\
 0.75 \times 2 \\
 0.50 \times 2
 \end{array}
 \quad
 \begin{array}{c|c}
 \text{Integer} & \text{Non-Integer} \\
 \hline
 0 & 0.75 \\
 1 & 0.50 \\
 1 & 0
 \end{array}
 \quad
 \begin{array}{c}
 \downarrow \\
 \downarrow
 \end{array}$$

Stop when Non-Integer = 0
Answer is Integers, in order.

$$0.375_{10} = 0.011_2$$

Ex: Convert 0.8125 to binary

| | <u>I</u> | <u>N</u> |
|-------------------|----------|---------------------|
| 0.8125×2 | 1 | $0.625 \rightarrow$ |
| 0.625×2 | 1 | 0.25 |
| 0.25×2 | 0 | 0.5 |
| 0.5×2 | 1 | 0 |

$$0.8125_{10} = 0.1101_2$$

Ex: Convert 0.734375 to hexadecimal

| | <u>I</u> | <u>N</u> |
|----------------------|----------|----------|
| 0.734375×16 | H B | 0.75 |
| 0.75×16 | 12 C | 0 |

$$0.734375 = 0.BC_{16}$$

$$\begin{aligned}10 &= A_{16} \\11 &= B_{16} \\12 &= C_{16}\end{aligned}$$

Ex: Repeating pattern

Convert 0.1 to binary

| | I | N |
|----------------|---|-----|
| 0.1×2 | 0 | 0.2 |
| 0.2×2 | 0 | 0.4 |
| 0.4×2 | 0 | 0.8 |
| 0.8×2 | 1 | 0.6 |
| 0.6×2 | 1 | 0.2 |
| 0.2×2 | | |

$$0.1 = 0.\overline{0011}_2$$

Ex: Convert 0.7 to octal

| | I | N |
|----------------|---|-----|
| 0.7×8 | 5 | 0.6 |
| 0.6×8 | 4 | 0.8 |
| 0.8×8 | 6 | 0.4 |
| 0.4×8 | 3 | 0.2 |
| 0.2×8 | 1 | 0.6 |

$$\left\{ \begin{array}{l} 0.6 \times 8 \\ 0.7 \end{array} \right.$$

$$0.7 = 0.5 \overline{4631}_8$$

Ex: Convert 19.96875 to octal

| | | | |
|---------------------------|---|---------------------------------------|--|
| $19 \div 8$ $2 \div 8$ | $\begin{array}{r} Q \quad R \\ \hline 2 & 3 \\ 0 & 2 \end{array}$ | 0.96875×8 0.75×8 | $\begin{array}{r} I \quad N \\ \hline 7 & 0.75 \\ 6 & 0 \end{array}$ |
|---------------------------|---|---------------------------------------|--|

$$19.96875 = 23.76_8$$

Ex: Convert 52.5625 to base 4

| | | | |
|--|---|--------------------------------------|--|
| $52 \div 4$ $13 \div 4$ $3 \div 4$ | $\begin{array}{r} Q \quad R \\ \hline 13 & 0 \\ 3 & 1 \\ 0 & 3 \end{array}$ | 0.5625×4 0.25×4 | $\begin{array}{r} I \quad N \\ \hline 2 & 0.25 \\ 1 & 0 \end{array}$ |
|--|---|--------------------------------------|--|

$$52.5625 = 310.21_4$$

1.3 Sugg HW #16

Convert to decimal (round to
3 decimal places)

$20C4.B7_{16}$

16^3 place ↑ 1 place 16^{-1} place

$$= 2 \times 16^3 + \cancel{C} \times 16^1 + 4 \times 16^0 + \cancel{B} \times 16^{-1} + 7 \times 16^{-2}$$

$$= 2 \times 4096 + 12 \times 16 + 4 + \frac{11}{16} + \frac{7}{16^2}$$

$$\approx 8388.715$$