Test 3

FRI Nov 10

9.2 – 9.8 (6 Questions)

Bring calculator

Bring music/earplugs

Practice Questions on website

9.7 Taylor Polynomials Cont'd

Ex: Find N so that the Maclaurin polynomial $P_N(x)$ approximates e^{-1} with error less than 0.001

$f(x) = e^x$

$f^{(n)}(x) = e^x$

$f^{(N+1)}(x) = e^x$

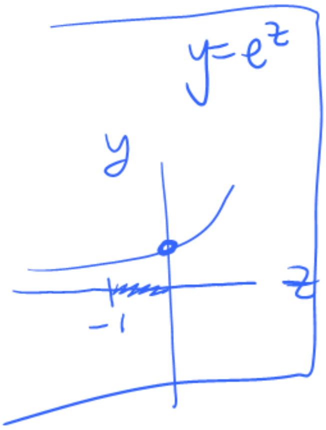
$f^{(N+1)}(z) = e^z$

$x = -1$

$c = 0$

$$|R_N(x)| = \left| \frac{f^{(N+1)}(z)}{(N+1)!} (x-c)^{N+1} \right|$$

$$= \left| \frac{e^z}{(N+1)!} (-1)^{N+1} \right|$$



$$= \left| \frac{e^z}{(N+1)!} \right|$$

where z is between -1 and 0

$$\leq \frac{e^0}{(N+1)!}$$

$$\leq \frac{1}{(N+1)!}$$

Want $\frac{1}{(N+1)!} < 0.001$

Guess and check.

$N=1$ ✗

$N=2$ ✗

$N=3$ ✗

$N=4$ ✗

$N=5$ ✗

$N=6$ ✓

$$\boxed{N \geq 6}$$

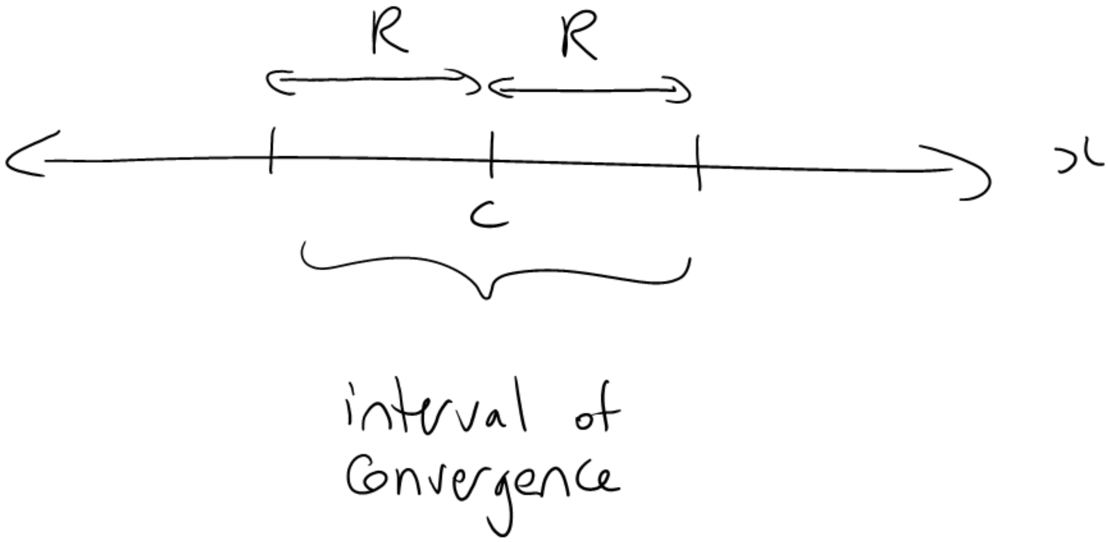
9.8 Power Series

Power series centred at c:

$$\sum_{n=0}^{\infty} b_n (x-c)^n = b_0 + b_1(x-c) + b_2(x-c)^2 + \dots$$

$b_0, b_1, b_2, \dots$ are the coefficients

See handout on website.



Ex: Find the interval of convergence:

$$\sum_{n=1}^{\infty} \underbrace{\frac{(-1)^n (x-3)^n}{n \cdot 5^n}}_{a_n}$$

Use ratio test.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{(x-3)^{n+1}}{(n+1)\cdot 5^{n+1}} \cdot \frac{n \cdot 5^n}{(x-3)^n} \right|$$

$$= \left| \frac{(x-3)}{5} \cdot \frac{n}{n+1} \right|$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x-3}{5}\cdot\frac{n}{n+1}\right|$$

$$= \left|\frac{x-3}{5}(1)\right|$$

l'Hôpital's Rule

$$= \left|\frac{x-3}{5}\right|$$

Series converges if $\left|\frac{x-3}{5}\right| < 1$

$$|x-3| < 5$$

$$-5 < x-3 < 5$$

$$-2 < x < 8$$

$x = -2$: $$\sum_{n=1}^{\infty}\frac{(-1)^n(-5)^n}{n\cdot 5^n} = \sum_{n=1}^{\infty}\frac{1\cancel{5^n}}{n\cdot\cancel{5^n}}$$

Diverges (p-series)

$x = 8$: $$\sum_{n=1}^{\infty}\frac{(-1)^n\cancel{5^n}}{n\cdot\cancel{5^n}}$$

Converges (Alternating Series Test)

$$\boxed{\text{Interval of Convergence: } -2 < x \le 8}$$