Test Mon Oct 23

8.2 - 8.5, 5.6, 8.8, 9.1

Review:

Ex: Evaluate $\lim_{x \to 0^+} (e^x + x)^{1/x}$

$$L = \lim_{x \to 0^+} (e^x + x)^{1/x}$$

$$\ln L = \lim_{x \to 0^+} \ln (e^x + x)^{1/x}$$

$$= \lim_{x \to 0^+} \frac{\ln(e^x + x)}{x} \leftarrow \frac{0}{0}$$

$$\overset{H}{=} \lim_{x \to 0^+} \frac{\frac{1}{e^x + x}(e^x + 1)}{1}$$

$$= 2$$

$$\ln L = 2 \Rightarrow L = e^{\ln L} = e^2.$$

ASIDE

$$\ln e^a = a \qquad e^{\ln b} = b$$

Ex: Evaluate, if possible:

$$\int_0^1 \frac{dx}{2x-1}$$

Asymptote at $x = \frac{1}{2}$

$$= \underbrace{\int_0^{1/2} \frac{dx}{2x-1}}_{I_1} + \underbrace{\int_{1/2}^{1} \frac{dx}{2x-1}}_{I_2}$$

$$I_1 = \int_0^{1/2} \frac{dx}{2x-1}$$

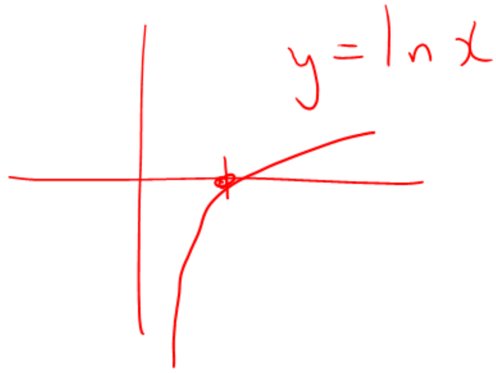
$$= \lim_{t \to \frac{1}{2}^-} \int_0^t \frac{dx}{2x-1}$$

$$= \lim_{t \to \frac{1}{2}^-} \frac{1}{2} \ln|2x-1| \Big|_0^t$$

$$= \lim_{t \to \frac{1}{2}^-} \frac{1}{2} \ln |2t-1| - \frac{1}{2} \ln 1$$

$$= \frac{1}{2} \ln 0^+$$

$$= -\infty$$



I_1 diverges

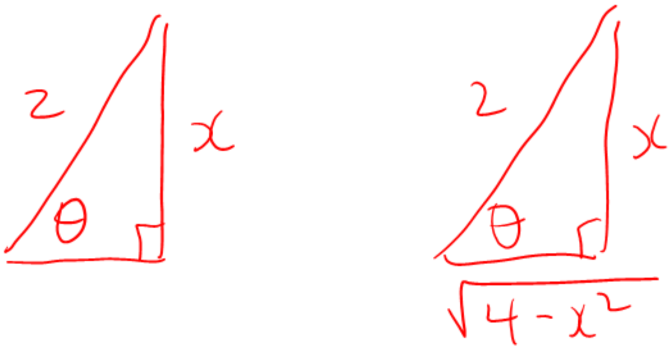
$\Rightarrow$ Integral diverges

Ex: Find $\int \sqrt{4-x^2}\, dx$

Trig Sub $\quad x = 2\sin\theta$

$$dx = 2\cos\theta\, d\theta$$

$$\frac{x}{2} = \sin\theta$$



$$\begin{cases} a^2 + x^2 = 4 \\ \vdots \\ a = \sqrt{4-x^2} \end{cases}$$

$$\boxed{\begin{aligned} \frac{\sqrt{4-x^2}}{2} &= \cos\theta \\ \sqrt{4-x^2} &= 2\cos\theta \end{aligned}}$$

$$\text{Integral} = \int 2\cos\theta\,(2\cos\theta\, d\theta)$$

$$= 4\int \cos^2\theta\, d\theta$$

$$= 2\int (1+\cos 2\theta)\, d\theta$$

$$= 2\left[\theta + \frac{\cancel{\sin 2\theta}^{\,2\sin\theta\cos\theta}}{2}\right] + C$$

$$= 2\left[\theta + \sin\theta\cos\theta\right] + C$$

$$= 2\left[\sin^{-1}\frac{x}{2} + \frac{x}{2}\frac{\sqrt{4-x^2}}{2}\right] + C \checkmark$$

$$= 2\sin^{-1}\frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} + C \checkmark$$

Ex: Find $\int \frac{4}{(x+3)(x^2+9)}\,dx$

$$\frac{4}{(x+3)(x^2+9)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+9}$$

$$4 = A(x^2+9) + (Bx+C)(x+3)$$

Sub $x=-3$: $\quad 4 = A(18) \Rightarrow A = \frac{2}{9}$

Coefficient of x^2: $\quad 0 = A + B \Rightarrow B = -\frac{2}{9}$

$x = 0$: $\quad 4 = 9A + 3C$

$$4 = 2 + 3C$$

$$2 = 3C$$

$$C = \frac{2}{3}$$

$$\text{Integral} = \int \left[\frac{2}{9}\frac{1}{x+3} - \frac{2}{9}\frac{x}{(x^2+9)} + \frac{2}{3}\frac{1}{(x^2+9)}\right] dx$$

$$= \frac{2}{9}\ln|x+3| - \frac{1}{9}\ln|x^2+9|$$

$$+ \quad \frac{2}{9}\tan^{-1}\frac{x}{3} + C$$

ASIDE

$$\int \frac{1}{4x^2+9}\,dx$$

$$= \int \frac{1}{(2x)^2+3^2}\,dx$$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$= \frac{1}{2}\int \frac{1}{u^2+3^2}\,du$$

$$= \frac{1}{6}\tan^{-1}\frac{u}{3} + C$$

Ex: $\int \cancel{\sin^3 2x}\cos^8 2x\,dx$

$\sin 2x\,(\sin^2 2x)$

$$= \int \sin 2x\,(1-\cos^2 2x)\cos^8 2x\,dx$$

$u = \cos 2x$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx$

$$= -\frac{1}{2}\int (1-u^2)\,u^8 \quad du$$

$$= -\frac{1}{2}\int (u^8 - u^{10})\,du$$

$$= -\frac{1}{2}\left[\frac{u^9}{9} - \frac{u^{11}}{11}\right] + C$$

$$= -\frac{1}{2}\left[\frac{\cos^9 2x}{9} - \frac{\cos^{11} 2x}{11}\right] + C$$

ASIDE

We can integrate:

$$\int < \text{powers of } \tan\theta > \sec^2\theta \, d\theta$$

$$\int < \text{powers of } \sec\theta > \sec\theta \tan\theta \, d\theta$$