9.5 Alternating Series

In an alternating series, signs alternate:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

Alternating Series Test

Let $a_n > 0$. The alternating series

$\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ both

converge if $\lim_{n\to\infty} a_n = 0$ and

$a_{n+1} \le a_n$ for all n.

Ex: Test for convergence:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$$

Alternating Series ✓

$$\lim_{n\to\infty} \frac{1}{2n-1} = 0 \quad ✓$$

a_n

$$a_{n+1} = \frac{1}{2(n+1)-1} = \frac{1}{2n+1}$$

$$\frac{1}{2n+1} \leq \frac{1}{2n-1} \quad \text{for all } n \; \checkmark$$

Series converges.

ASIDE If $a_{n+1} > a_n$ then the Alternating Series Test cannot be applied.

Recall

partial sum S_N = sum of first N terms

S = sum of series

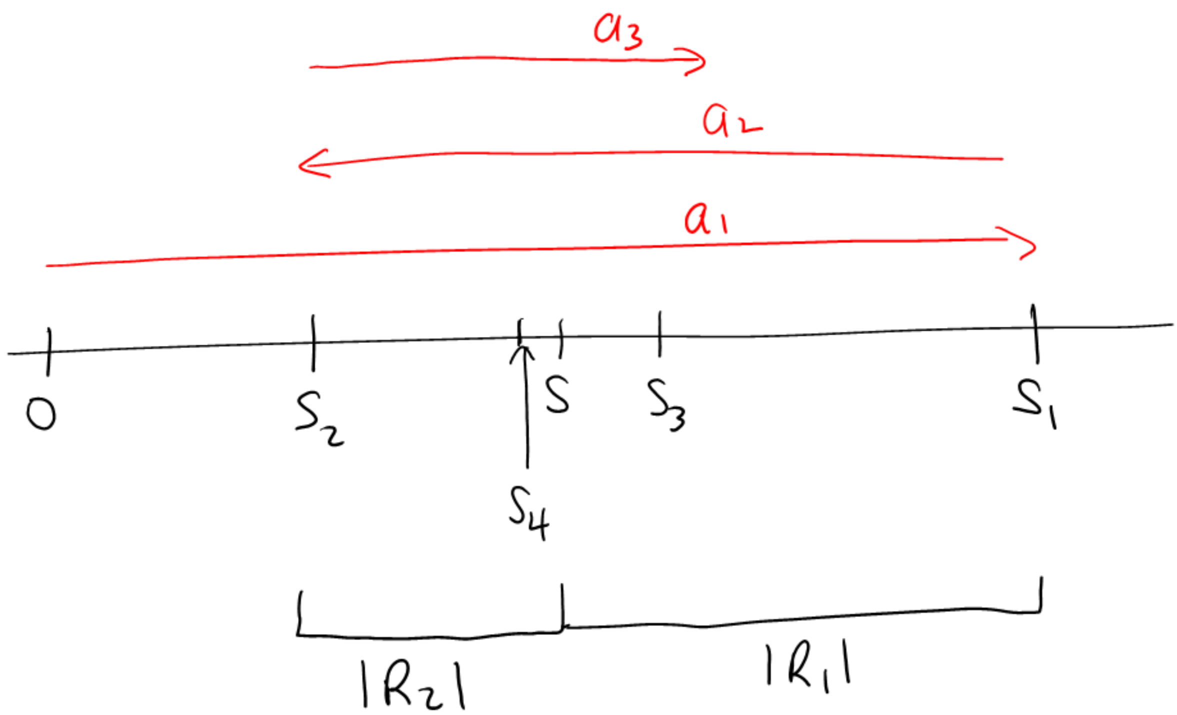
remainder/error $R_N = S - S_N$

FACT

If a series converges by the Alternating Series Test

then $|R_N| \leq a_{N+1}$

Why?



$$|R_1| \leq a_2$$

$$|R_2| \leq a_3$$

$$|R_3| \leq a_4$$

etc.

Ex: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges by the Alternating Series Test.

a) Calculate S_{19}

$$S_{19} = 1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{1}{19}$$

$$\approx 0.7188$$

b) Find an upper bound for $|R_{19}|$

$$|R_{19}| \leq a_{20}$$

$$\leq \frac{1}{20}$$

$$\leq 0.05$$

c) Estimate S

$$0.7188 - 0.05 \leq S \leq 0.7188 + 0.05$$

ASIDE: If we had used S_4, we would have a wider interval for possible S-values.

Preview: Find N so that $|R_N| \le 0.005$ given that the series converges by the A.S.T.

Bigger $N \Rightarrow$ more terms in S_N

$\Rightarrow$ smaller $|R_N|$