Final Exam
Thurs Dec 14
1:30pm - 4:30pm
TEC 175

Test 2 Mon Oct 23 ~~Fri Oct 20~~
Test 4 Mon Dec 4 ~~Fri Dec 1~~
Test 3 Fri Nov 10

9.3 The Integral Test and p-Series Cont'd

Def

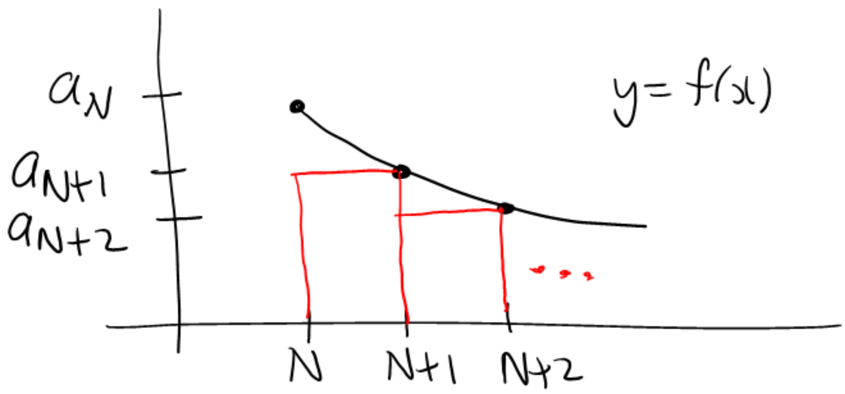
$$\sum_{n=1}^{\infty} a_n = \underbrace{\sum_{n=1}^{N} a_n}_{\substack{\text{partial sum} \\ S_N}} + \underbrace{\sum_{n=N+1}^{\infty} a_n}_{\substack{\text{remainder} \\ \text{or error} \\ R_N}}$$

Fact

Suppose $\sum_{n=1}^{\infty} a_n$ converges by the Integral Test.

Then $R_N \leq \int_N^{\infty} f(x)\,dx$

Why?



$$R_N \le \int_N^\infty f(x)\,dx$$

sum of areas of red rectangles

area under curve

Ex: $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by the Integral Test.

a) Find S_{10}

$$S_{10} = 1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \frac{1}{216} + \frac{1}{343} + \frac{1}{512} + \frac{1}{729} + \frac{1}{1000}$$

$$\approx 1.19753199$$

b) Find an upper bound for the error R_{10}.

$$R_{10} \le \int_{10}^{\infty} \frac{1}{x^3}\,dx$$

$$\le \lim_{b\to\infty} \int_{10}^{b} x^{-3}\,dx$$

$$\le \lim_{b\to\infty} -\frac{1}{2}x^{-2} \Big|_{10}^{b}$$

$$\le \lim_{b\to\infty} -\frac{1}{2}b^{-2} + \frac{1}{2}(10)^{-2}$$

$$\leq 0.005$$

c) Estimate $\sum_{n=1}^{\infty} \frac{1}{n^3}$ using S_{10} and R_{10}.

$$S_{10} \leq \sum_{n=1}^{\infty} \frac{1}{n^3} \leq S_{10} + R_{10}$$

$$1.19753199 \leq \sum_{n=1}^{\infty} \frac{1}{n^3} \leq 1.19753199 + 0.005$$

d) Find N so that

$$R_N \leq 0.0005$$

see next page →

Plan: Calculate $\int_N^\infty f(x)\,dx$

Let $\int_N^\infty f(x)\,dx \le 0.0005$

$$\int_N^\infty \frac{1}{x^3}\,dx = \lim_{b\to\infty} \int_N^b x^{-3}\,dx$$

$$= \lim_{b\to\infty} -\frac{1}{2}x^{-2}\Big|_N^b$$

$$= \lim_{b\to\infty} -\frac{1}{2}b^{-2} + \frac{1}{2}N^{-2}$$

$$= \frac{1}{2}N^{-2}$$

$$\frac{1}{2}N^{-2} \le 0.0005$$

$$\frac{1}{2} \le 0.0005 N^2$$

$$\frac{1}{2(0.0005)} \le N^2$$

Take square roots:

$$\sqrt{\frac{1}{2(0.0005)}} \leq N$$

$$N \geq 31.6$$

$$\boxed{N \geq 32}$$