Test 2

Mon Oct 23

8.2 - 8.5, 5.6, 8.8, 9.1 (7 Questions)

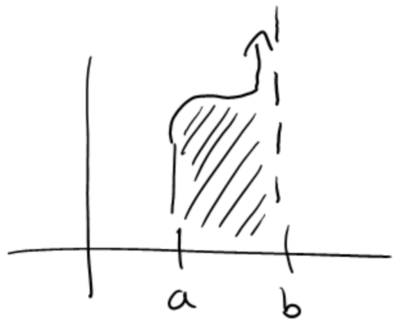
Bring: calculator, music/earplugs

Practice Problems on website

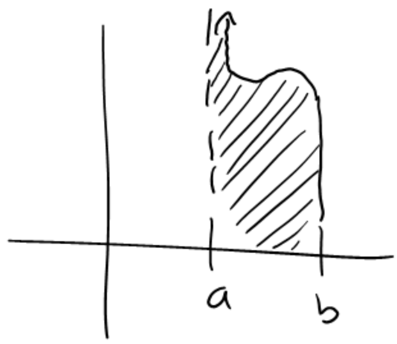
8.8 Cont'd

See handout (page two)

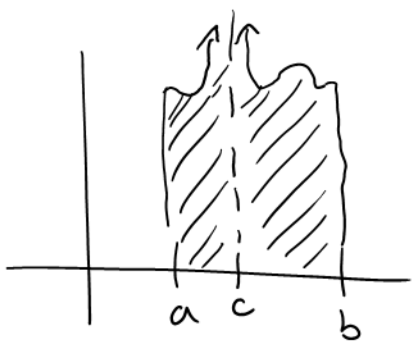
Picture for Fact 4:



Picture for Fact 5:



Picture for Fact 6



Ex: Evaluate or show that it diverges:

$$\int_2^5 \frac{1}{\sqrt{x-2}}\,dx$$

$$= \lim_{t\to 2^+} \int_t^5 (x-2)^{-1/2}\,dx$$

$$= \lim_{t\to 2^+} 2(x-2)^{1/2} \Big|_t^5$$

$$= \lim_{t\to 2^+} 2\sqrt{3} - 2\sqrt{t-2}$$

$$= 2\sqrt{3} - 0$$

$$= 2\sqrt{3}$$

Ex: Evaluate, if possible.

$$\int_0^3 \frac{1}{(x-3)^2}\,dx$$

$$= \lim_{t\to 3^-} \int_0^t (x-3)^{-2}\,dx$$

$$= \lim_{t\to 3^-} -(x-3)^{-1} \Big|_0^t$$

$$= \lim_{t \to 3^-} \frac{-1}{t-3} + \frac{1}{-3}$$

$$= \infty - \frac{1}{3}$$

$$= \infty$$

The integral diverges.

Ex: Evaluate, if possible:

$$\int_0^3 \frac{1}{x-1}\,dx$$

Caution: Asymptote at $x=1$

$$= \underbrace{\int_0^1 \frac{1}{x-1}\,dx}_{I_1} + \underbrace{\int_1^3 \frac{1}{x-1}\,dx}_{I_2}$$

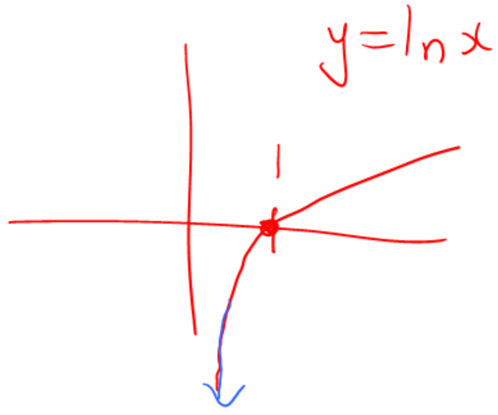
$$I_1 = \int_0^1 \frac{1}{x-1}\,dx$$

$$= \lim_{t \to 1^-} \int_0^t \frac{1}{x-1}\,dx$$

$$= \lim_{t \to 1^-} \ln|x-1| \Big|_0^t$$

$$= \lim_{t\to 1^-} \ln|t-1| - \underbrace{\cancel{\ln 1}}_{0}$$

$$= -\infty$$



I_1 diverges

$\Rightarrow$ Integral diverges

Extra: Evaluate, if possible:

$$\int_0^3 (x-2)^{-2/3}\,dx$$

Caution: Asymptote at $x = 2$

$$\text{Integral} = \underbrace{\int_0^2 (x-2)^{-2/3}\,dx}_{I_1} + \underbrace{\int_2^3 (x-2)^{-2/3}\,dx}_{I_2}$$

$$I_1 = \int_0^{2} (x-2)^{-2/3}\,dx$$

$$= \lim_{t\to 2^-} \int_0^t (x-2)^{-2/3}\,dx$$

$$= \lim_{t\to 2^-} 3(x-2)^{1/3} \Big|_0^t$$

$$= \lim_{t\to 2^-} 3(t-2)^{1/3} - 3(-2)^{1/3}$$

$$= \quad 0 \quad + 3(2)^{1/3}$$

$$= 3(2)^{1/3}$$

$$I_2 = \int_2^3 (x-2)^{-2/3}\,dx$$

$$= \lim_{t\to 2^+} \int_t^3 (x-2)^{-2/3}\,dx$$

$$\vdots$$

$$= 3$$

$$\text{Integral} = I_1 + I_2$$

$$= 3(2)^{1/3} + 3$$

$\sqrt{-1}$ not a real #

$\sqrt[4]{-7}$ ״

$$\boxed{\begin{aligned} (-1)^2 &= 1 \\ (-1)^3 &= -1 \\ (-8)^{1/3} &= -2 \\ \sqrt[3]{-2} &= -\sqrt[3]{2} \end{aligned}}$$

9.1 Sequences

A <u>sequence</u> is an infinite ordered list of numbers.

Notation:

$$a_0, a_1, a_2, \dots$$

$$\{a_n\}_{n=0}^{\infty}$$

A sequence could begin at $n=0$, $n=1$ or any other value.

<u>Ex</u>: Find the first 3 terms:

$$\left\{\frac{(-1)^n}{2^n}\right\}_{n=0}^{\infty}$$

$$1, \frac{-1}{2}, \frac{1}{4}, \dots$$

<u>Ex</u>: $a_0 = 0$, $a_1 = 1$

$a_{n+2} = a_{n+1} + a_n$ for $n \geq 0$

Find the next 3 terms.

$(n=0)$ $a_2 = a_1 + a_0 = 1$

$(n=1)$ $a_3 = a_2 + a_1 = 2$

$(n=2)$ $a_4 = a_3 + a_2 = 3$

If $\lim_{n\to\infty} a_n$ exists and is a real number then the sequence <u>converges</u>. Otherwise the sequence <u>diverges</u>.

<u>Ex</u>: Find the sequence's limit, if possible.

a) $\left\{\frac{1}{3n+1}\right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{1}{3n+1}$$

$$= 0$$

Sequence converges to 0.

b) $\{n^2\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} n^2$$

$$= \infty$$

Sequence diverges.

c) $\{(-1)^n\}_{n=1}^{\infty}$

$\lim_{n\to\infty} (-1)^n$ does not exist (oscillates)

Sequence diverges.

d) $\left\{ \frac{3n}{\sqrt{n^2+1}} \right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{3n}{\sqrt{n^2+1}}$$

$$= \lim_{n\to\infty} 3\cdot\sqrt{\frac{n^2}{n^2+1}}$$

$$= \lim_{n\to\infty} 3\cdot\sqrt{\frac{1}{1+\frac{1}{n^2}}} \qquad \frac{1}{n^2} \to 0$$

$$= 3$$

Sequence converges to 3.

We can use l'Hôpital's Rule on sequences.

Ex: Find the limit of $\left\{ \frac{2n}{3n+1} \right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{2n}{3n+1} \qquad \frac{\infty}{\infty}$$

$$\overset{H}{=} \lim_{n\to\infty} \frac{2}{3}$$

$$= \frac{2}{3}$$

Sequence converges to $\frac{2}{3}$.

Squeeze Theorem

If $a_n \le b_n \le c_n$ for all n

and $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n$

then $\lim_{n\to\infty} b_n = L$.

Ex: Find the limit of $\left\{ \frac{\cos n}{n} \right\}_{n=1}^{\infty}$

$$\frac{-1}{n} \le \frac{\cos n}{n} \le \frac{1}{n} \quad \text{for all } n$$

$$\lim_{n\to\infty} \frac{-1}{n} = 0 = \lim_{n\to\infty} \frac{1}{n}$$

Therefore $\lim_{n\to\infty} \frac{\cos n}{n} = 0$.

The sequence converges to 0.

$$n! = n(n-1)(n-2)\cdots 1$$

"n factorial"

$$5! = 5(4)(3)(2)(1) = 120$$

$$2! = 2(1) = 2$$

$$1! = 1$$

$$0! = 1 \qquad \text{by definition}$$

<u>Ex</u>: Simplify $\dfrac{(n+1)!}{(n-1)!}$

$$= \frac{(n+1)(n)\cancel{(n-1)(n-2)\cdots 1}}{\cancel{(n-1)(n-2)\cdots 1}}$$

$$= (n+1)n$$