

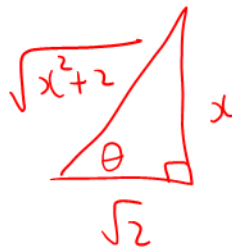
8.4 Trig Substitution Cont'd

Ex: Find $\int \frac{dx}{\sqrt{x^2+2}}^3$

$$x = \sqrt{2} \tan \theta$$

$$dx = \sqrt{2} \sec^2 \theta d\theta$$

$$\frac{x}{\sqrt{2}} = \tan \theta$$



$$\begin{cases} a^2 + b^2 = c^2 \\ 2 + x^2 = c^2 \\ c = \sqrt{x^2 + 2} \end{cases}$$

desired root
#

$$\frac{\sqrt{x^2+2}}{\sqrt{2}} = \sec \theta$$

$$\sqrt{x^2+2} = \sqrt{2} \sec \theta$$

$$\text{Integral} = \int \frac{\sqrt{2} \sec^2 \theta d\theta}{(\sqrt{2} \sec \theta)^3}$$

$$= \frac{1}{2} \int \frac{1}{\sec \theta} d\theta$$

$$= \frac{1}{2} \int \cos \theta d\theta$$

$$= \frac{1}{2} \sin \theta + C$$

$$= \frac{1}{2} \frac{x}{\sqrt{x^2+2}} + C$$

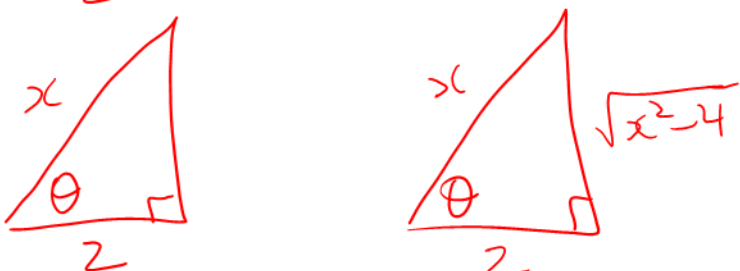
Ex:

Find

$$\int_3^5 \frac{dx}{x^2 \sqrt{x^2-4}}$$

$$x = 2 \sec \theta$$

$$dx = 2 \sec \theta \tan \theta d\theta$$

$$\frac{x}{2} = \sec \theta$$

$$\frac{\sqrt{x^2-4}}{2} = \tan \theta$$
$$\sqrt{x^2-4} = 2 \tan \theta$$

$$\text{Integral} = \int_{x=3}^{x=5} \frac{2 \sec \theta \tan \theta d\theta}{(2 \sec \theta)^2 (2 \tan \theta)}$$

$$= \frac{1}{4} \int_{x=3}^{x=5} \frac{1}{\sec \theta} d\theta$$

$$= \frac{1}{4} \int_{x=3}^{x=5} \cos \theta d\theta$$

$$= \frac{1}{4} \left[\sin \theta \right]_{x=3}^{x=5}$$

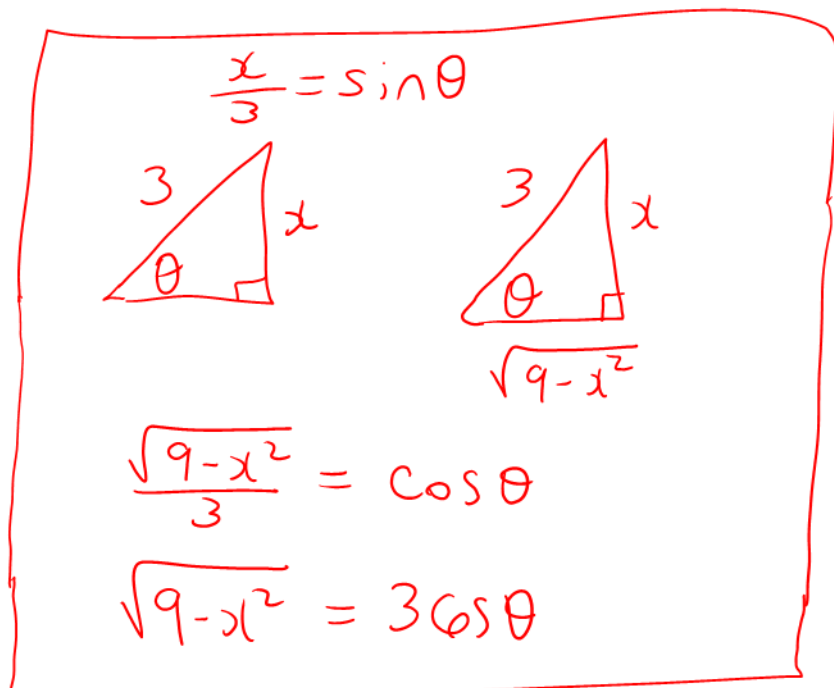
$$= \frac{1}{4} \left[\frac{\sqrt{x^2-4}}{x} \right]_3^5$$

$$= \frac{1}{4} \left[\frac{\sqrt{21}}{5} - \frac{\sqrt{5}}{3} \right]$$

Ex: Find $\int \sqrt{9-x^2} dx$

$$x = 3 \sin \theta$$

$$dx = 3 \cos \theta d\theta$$



$$\text{Integral} = \int 3 \cos \theta (3 \cos \theta d\theta)$$

$$= 9 \int \cos^2 \theta d\theta$$

$$= \frac{9}{2} \int (1 + \cos 2\theta) d\theta$$

$$= \frac{9}{2} \left[\theta + \frac{\sin 2\theta}{2} \right] + C$$

Section 8.3
 $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{9}{2} [\theta + \sin\theta\cos\theta] + C$$

$$= \frac{9}{2} \left[\sin^{-1} \frac{x}{3} + \frac{x}{3} \frac{\sqrt{9-x^2}}{3} \right] + C$$

Ex: Find $\int \frac{dx}{\sqrt{4x^2+9}}$

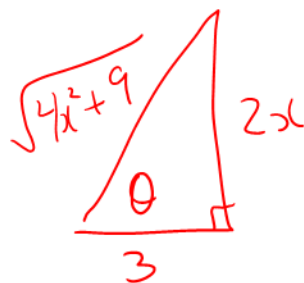
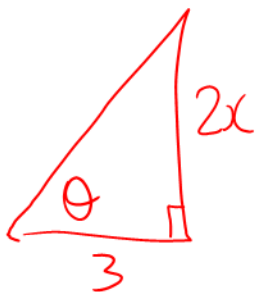
$$= \int \frac{dx}{\sqrt{(2x)^2+3^2}}$$

$$2x = 3 \tan \theta$$

$$x = \frac{3}{2} \tan \theta$$

$$dx = \frac{3}{2} \sec^2 \theta d\theta$$

$$\frac{2x}{3} = \tan \theta$$



$$\frac{\sqrt{4x^2+9}}{3} = \sec \theta$$

$$\sqrt{4x^2+9} = 3 \sec \theta$$

$$\text{Integral} = \frac{3}{2} \int \frac{\sec^2 \theta d\theta}{3 \sec \theta}$$

$$= \frac{1}{2} \int \sec \theta d\theta$$

$$= \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$$

$$= \frac{1}{2} \ln \left| \frac{\sqrt{4x^2+9}}{3} + \frac{2x}{3} \right| + C$$

8.5 Partial Fractions

$$\frac{7x+1}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

"partial fraction expansion"

Multiply by $(x+3)(x-1)$:

$$7x+1 = \frac{A}{\cancel{(x+3)}} \cancel{(x+3)}(x-1) + B(x+3)$$

$$\boxed{7x+1 = A(x-1) + B(x+3)}$$

Method 1: SUBSTITUTE

$$x=1:$$

$$8 = B(4) \Rightarrow B=2$$

$$x = -3:$$

$$-20 = A(-4) \Rightarrow A = 5$$

Method 2: MATCH COEFFICIENTS

$$7x+1 = Ax - A + Bx + 3B$$

$$7x+1 = (A+B)x + (-A+3B)$$



$$\begin{cases} A+B = 7 \\ -A+3B = 1 \end{cases}$$

$$B = 2$$

$$A = 5$$

Conclude $\frac{7x+1}{(x+3)(x-1)} = \frac{5}{x+3} + \frac{2}{x-1}$

Shortcut

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln |ax+b| + C$$

Quick Ex:

$$\int \frac{7x+1}{(x+3)(x-1)} dx$$

⋮

$$= \int \left[\frac{5}{x+3} + \frac{2}{x-1} \right] dx$$

$$= 5 \ln |x+3| + 2 \ln |x-1| + C$$