Test Review

Ex: Find $\lim\limits_{x\to 0} \dfrac{\sqrt{x+5}-\sqrt{5}}{x} \cdot \dfrac{(\sqrt{x+5}+\sqrt{5})}{(\sqrt{x+5}+\sqrt{5})}$

$$= \lim_{x\to 0} \frac{x+5-5}{x(\sqrt{x+5}+\sqrt{5})}$$

$$= \lim_{x\to 0} \frac{\cancel{x}\ 1}{\cancel{x}(\sqrt{x+5}+\sqrt{5})}$$

$$= \frac{1}{2\sqrt{5}}$$

$\dfrac{+}{0^+} = +\infty$ $\qquad$ $\dfrac{-}{0^+} = -\infty$

$\dfrac{0}{0}$: no info

Ex: Find $\dfrac{dy}{dx}$ given

$$(x^2)(y^2) - 9x^2 - 4y^2 = 0$$

Take $\dfrac{d}{dx}$:

$$x^2\left(2y\frac{dy}{dx}\right)+y^2(2x)-18x-8y\frac{dy}{dx}=0$$

Solve for $\frac{dy}{dx}$:

$$2x^2y\frac{dy}{dx}-8y\frac{dy}{dx}=18x-2xy^2$$

$$\left[2x^2y-8y\right]\frac{dy}{dx}=18x-2xy^2$$

$$\frac{dy}{dx}=\frac{18x-2xy^2}{2x^2y-8y}$$

$\frac{d}{dx}$: take the derivative

$$\frac{d}{dx}(x^3)=3x^2$$

$$\frac{d}{dx}(y^3)=3y^2\frac{dy}{dx}$$

the deriv. of y

$$\frac{d}{dx}(e^y+\ln y)=e^y\frac{dy}{dx}+\frac{1}{y}\frac{dy}{dx}$$

Ex: Find $f'(x)$

a) $f(x) = 6\arctan x^8 - 2\arcsin x^7 + x^3\sec^2 x$

$$f'(x) = 6 \cdot \frac{1}{1+(x^8)^2} \cdot 8x^7 - 2 \cdot \frac{1}{\sqrt{1-(x^7)^2}} \cdot 7x^6$$

$$+ x^3 \left[2\sec x \sec x \tan x\right] + (\sec^2 x)(3x^2)$$

$$= \frac{48x^7}{1+x^{16}} - \frac{14x^6}{\sqrt{1-x^{14}}}$$

$$+ 2x^3\sec^2 x \tan x + 3x^2\sec^2 x$$

$$\boxed{\begin{aligned} &\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C \\ &\frac{d}{dx}\left[\frac{1}{a}\arctan\frac{x}{a}\right] = \frac{1}{a^2+x^2} \\ &\frac{d}{dx}\left[\arctan x\right] = \frac{1}{1+x^2} \end{aligned}}$$

b) $f(x) = \dfrac{e^{4x}}{\ln(2x+3) - 2}$

$$f'(x) = \frac{[\ln(2x+3)-2](4e^{4x}) - e^{4x}\cdot\frac{1}{2x+3}(2)}{[\ln(2x+3)-2]^2}$$

$$= \frac{4e^{4x}[\ln(2x+3)-2] - \frac{2e^{4x}}{2x+3}}{[\ln(2x+3)-2]^2}$$

$$\boxed{\begin{gathered}\int e^x\,dx = e^x + c \\ \frac{d}{dx}e^x = e^x\end{gathered}}$$

$$\boxed{\begin{gathered}\int \frac{1}{x}\,dx = \ln|x| + C \\ \frac{d}{dx}\ln x = \frac{1}{x}\end{gathered}}$$

Ex: Find $\int \frac{1}{\sqrt{x}(1+3\sqrt{x})}dx$

$$\boxed{\begin{gathered}u = 1+3\sqrt{x} \\ du = \frac{3}{2}x^{-1/2}dx \\ \frac{2}{3}du = \frac{dx}{\sqrt{x}}\end{gathered}}$$

$$= \frac{2}{3}\int\frac{du}{u}$$

$$= \frac{2}{3}\ln|u| + C$$

$$= \frac{2}{3}\ln|1+3\sqrt{x}| + C$$

Ex: $\int \frac{1-\sin x}{\cos x}\,dx$

$$= \int \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) dx$$

$$= \int (\sec x - \tan x)\,dx$$

$$= \ln|\sec x + \tan x| - \ln|\sec x| + C$$

Ex: $\int \cos 4x \sqrt{1+\sin 4x}\,dx$

$$\boxed{\begin{aligned} u &= 1+\sin 4x \\ du &= 4\cos 4x\,dx \\ \frac{du}{4} &= \cos 4x\,dx \end{aligned}}$$

$$= \frac{1}{4}\int \sqrt{u}\,du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2}\right] + C$$

$$= \frac{1}{6}(1+\sin 4x)^{3/2} + C$$

Ex: $\int e^{\cos 2x} \sin 2x\,dx$

$$\begin{aligned} u &= \cos 2x \\ du &= -2\sin 2x\,dx \end{aligned}$$

$$-\frac{1}{2}du = \sin 2x\,dx$$

$$= -\frac{1}{2}\int e^{u}\,du$$

$$= -\frac{1}{2}e^{u} + C$$

$$= -\frac{1}{2}e^{\cos 2x} + C$$

Ex: $\displaystyle\int \frac{x^2-1}{x+3}\,dx$

$$\begin{array}{r l}
 & x - 3 \\
(x+3) & \overline{\big)\, x^2 + 0x - 1} \\
 & -(x^2 + 3x) \\
\hline
 & -3x - 1 \\
 & -(-3x - 9) \\
\hline
 & 8
\end{array}$$

$$\text{Integral} = \int \left[x - 3 + \frac{8}{x+3}\right]dx$$

$$= \frac{x^2}{2} - 3x + 8\ln|x+3| + C$$

Ex: $\displaystyle\int \frac{e^{x}}{\sqrt{3-e^{2x}}}\,dx$

$$= \int \frac{e^x}{\sqrt{\sqrt{3}^2 - (e^x)^2}}\, dx$$

$$\boxed{\begin{aligned} u &= e^x \\ du &= e^x\, dx \end{aligned}}$$

$$= \int \frac{du}{\sqrt{\sqrt{3}^2 - u^2}}$$

$$= \sin^{-1} \frac{u}{\sqrt{3}} + C$$

$$= \sin^{-1} \frac{e^x}{\sqrt{3}} + C$$

$$\boxed{\int \frac{dx}{ax+b} = \frac{1}{a} \ln |ax+b| + C}$$

$$\int \frac{dx}{x+4} = \ln|x+4| + C$$

$$\int \frac{dx}{3x+4} = \frac{1}{3} \ln|3x+4| + C$$

Q: $y = x^3 + 1$

Find the equation of the tangent line at $(2, 9)$.

$$y' = 3x^2$$

$$m = 3x^2 \Big|_{x=2}$$

$$= 12$$

$$y - y_1 = m(x - x_1)$$

$$y - 9 = 12(x - 2)$$