# 8.3 Trig Integrals Cont'd

Three other Techniques

1) Integration By Parts
2) Convert to $\sin\theta$ and $\cos\theta$
3) Identities

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$$

Ex: $\int \sec^3\theta \, d\theta$

$$\boxed{\begin{array}{ll} u = \sec\theta & dv = \sec^2\theta \, d\theta \\ du = \sec\theta\tan\theta & v = \tan\theta \end{array}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$$

Let $I = \int \sec^3\theta\,d\theta$

$$I = \sec\theta\tan\theta - I + \ln|\sec\theta + \tan\theta|$$

$$2I = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + C_1$$

$$I = \frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right] + C_2$$

Ex: $\int \sec\theta\cot^2\theta\,d\theta$

$$= \int \frac{1}{\cos\theta}\left(\frac{\cos\theta}{\sin\theta}\right)^2 d\theta$$

$$= \int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\, d\theta \end{aligned}}$$

$$= \int \frac{du}{u^2}$$

$$= \int u^{-2}\, du$$

$$= -u^{-1} + C$$

$$= -\csc\theta + C$$

Ex: $\int \sin 3x \cos 7x \, dx$

$$= \frac{1}{2}\int \left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right] dx$$

$$= \frac{1}{2}\int \left[\sin(-4x) + \sin(10x)\right] dx$$

$$= \frac{1}{2}\left[-\frac{\cos(-4x)}{-4} - \frac{\cos 10x}{10}\right] + C$$

$$= \frac{1}{2}\left[\frac{\cos(-4x)}{4} - \frac{\cos 10x}{10}\right] + C$$

or $\frac{1}{2}\left[\frac{\cos 4x}{4}-\frac{\cos 10x}{10}\right]+C$

$\cos(-100^\circ)=\cos 100^\circ$

$\sin(-100^\circ)=-\sin 100^\circ$

## 8.4 Trig Substitution

$\sin\theta=\frac{O}{H}$ $\cos\theta=\frac{A}{H}$ $\tan\theta=\frac{O}{A}$

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$\csc\theta=\frac{H}{O}$ $\sec\theta=\frac{H}{A}$ $\cot\theta=\frac{A}{O}$

| If integral contains | then substitute |
|---|---|
| $\sqrt{a^2-x^2}$ | $x=a\sin\theta$ |
| $\sqrt{a^2+x^2}$ | $x=a\tan\theta$ |
| $\sqrt{x^2-a^2}$ | $x=a\sec\theta$ |

$(a>0)$

Ex:

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$x = 2\sin\theta$$

$$dx = 2\cos\theta\, d\theta$$

$$\frac{x}{2} = \sin\theta$$



$$a^2 + b^2 = c^2$$

$$a^2 + x^2 = 4$$

$$a^2 = 4 - x^2$$

$$a = \sqrt{4-x^2}$$

$$\frac{\sqrt{4-x^2}}{2} = \cos\theta$$

$$\sqrt{4-x^2} = 2\cos\theta$$

$$I = \int \frac{2\cos\theta \, d\theta}{(2\sin\theta)^2 (2\cos\theta)}$$

$$= \frac{1}{4} \int \csc^2\theta \, d\theta$$

$$= -\frac{1}{4} \cot\theta + C$$

$$= -\frac{1}{4} \frac{\sqrt{4-x^2}}{x} + C$$