Test 1
FRI SEPT 29
Bring calculator
Bring music/earplugs
Practice Problems on website

Covers:

1.2-1.5
2.2-2.4
5.1
2.5
4.4-4.5
5.2, 5.4
5.7
5.8
8.1

8.3 Trig Integrals Cont'd

Recap: a) $\int \sin^4\theta \cos\theta \, d\theta$

$u = \sin\theta$
$du = \cos\theta \, d\theta$

b) $\int \sin^4\theta \cos^3\theta \, d\theta$

$= \int \sin^4\theta (1-\sin^2\theta) \cos\theta \, d\theta$

$u = \sin\theta$
$du = \cos\theta \, d\theta$

To integrate even powers:

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Ex: $\int \cos^2\theta \, d\theta$

$$= \int \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

Ex: $\int \sin^4 3x \, dx$

$$= \int (\sin^2 3x)^2 \, dx$$

$$= \int \left[\frac{1-\cos 6x}{2}\right]^2 dx$$

$$= \frac{1}{4}\int (1 - 2\cos 6x + \cos^2 6x)\, dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos 6x + \frac{1}{2} + \frac{\cos 12x}{2}\right) dx$$

$$= \frac{1}{4}\left[x - \frac{\sin 6x}{3} + \frac{x}{2} + \frac{\sin 12x}{24}\right] + C$$

To evaluate $\int \sec^m\theta \tan^n\theta\, d\theta$:

$$1 + \tan^2\theta = \sec^2\theta$$

Sub $u = \tan\theta$ and $du = \sec^2\theta\, d\theta$

OR

$u = \sec\theta$ and $du = \sec\theta\tan\theta\, d\theta$

Ex: $\int \tan^3\theta \sec^3\theta\, d\theta$

$= \int \tan^2\theta \sec^2\theta \sec\theta\tan\theta\, d\theta$

$(\sec^2\theta - 1)$

$u = \sec\theta$
$du = \sec\theta\tan\theta\, d\theta$

$= \int (u^2 - 1)u^2\, du$

$= \int (u^4 - u^2)\, du$

$= \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \frac{1}{5}\sec^5\theta - \frac{1}{3}\sec^3\theta + C$

Ex: $\int \tan^4\theta\, d\theta$

$= \int \tan^2\theta \cancel{\tan^2\theta}\, d\theta$
$\quad (\sec^2\theta - 1)$

$= \int (\tan^2\theta \sec^2\theta - \tan^2\theta)\, d\theta$

$= \int [\tan^2\theta \sec^2\theta - (\sec^2\theta - 1)]\, d\theta$

$= \int \underbrace{\tan^2\theta \sec^2\theta\, d\theta - \int \sec^2\theta\, d\theta} + \int 1\, d\theta$

$u = \tan\theta$
$du = \sec^2\theta\, d\theta$

$= \int u^2\, du - \int du + \theta + C$

$= \frac{u^3}{3} - u + \theta + C$

$= \frac{1}{3}\tan^3\theta - \tan\theta + \theta + C$

To evaluate $\int \csc^m\theta \cot^n\theta\, d\theta$:

$1 + \cot^2\theta = \csc^2\theta$

Sub $u = \csc\theta$ $\quad du = -\csc\theta\cot\theta\, d\theta$

OR

$u = \cot\theta$ $\quad du = -\csc^2\theta\, d\theta$

Ex: $\int \cot^2 3x \csc^4 3x\, dx$

$= \int \cot^2 3x \csc^2 3x \csc^2 3x\, dx$

$(1+\cot^2 3x)$

$u = \cot 3x$

$du = -3\csc^2 3x\, dx$

$-\frac{1}{3} du = \csc^2 3x\, dx$

$= -\frac{1}{3}\int u^2(1+u^2)\, du$

$= -\frac{1}{3}\int (u^2+u^4)\, du$

$= -\frac{1}{3}\left[\frac{u^3}{3} + \frac{u^5}{5}\right] + C$

$$= -\frac{1}{3}\left[\frac{1}{3}\cot^3 3x + \frac{1}{5}\cot^5 3x\right] + C$$