8.2 Integration By Parts Cont'd

$$\int u\,dv = uv - \int v\,du$$

<u>Ex:</u> $\int \arctan x\,dx$

$u = \arctan x \qquad dv = dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x$

Don't write $+C$

$$\int u\,dv = uv - \int v\,du$$

$$\int \arctan x\,dx = x\arctan x - \int \frac{x}{1+x^2}\,dx$$

$w = 1+x^2$
$dw = 2x\,dx$
$\frac{dw}{2} = x\,dx$

$$= x\arctan x - \frac{1}{2}\int \frac{dw}{w}$$

$$= x\arctan x - \frac{1}{2}\ln|w| + C$$
$$= x\arctan x - \frac{1}{2}\ln|1+x^2| + C$$

Ex: $\int x\sqrt{1+x}\,dx$

$$\boxed{\begin{array}{ll} u = x & dv = (1+x)^{1/2}\,dx \\ du = dx & v = \frac{2}{3}(1+x)^{3/2} \end{array}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\sqrt{1+x}\,dx = \frac{2}{3}x(1+x)^{3/2} - \int \frac{2}{3}(1+x)^{3/2}\,dx$$

$$= \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(1+x)^{5/2} + C$$

$$= \frac{2}{3}x(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} + C$$

Ex: $\int e^{2x}\cos x\,dx$

$$u = e^{2x} \qquad dv = \cos x\,dx$$
$$du = 2e^{2x}\,dx \qquad v = \sin x$$

$$\int u\,dv = uv - \int v\,du \qquad (1)$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\underbrace{\int e^{2x}\sin x\,dx}_{\text{Integration By Parts}}$$

$$\int e^{2x}\sin x\,dx = ?$$

$$u = e^{2x} \qquad dv = \sin x\,dx$$
$$du = 2e^{2x}\,dx \qquad v = -\cos x$$

$$\int u\,dv = uv - \int v\,du \qquad (2)$$

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2\int e^{2x}\cos x\,dx$$

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$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\left[-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx\right]$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x\,dx$$

$$\text{let } I = \int e^{2x}\cos x\,dx$$

$$I = e^{2x}\sin x + 2e^{2x}\cos x - 4I$$

$$5I = e^{2x}(\sin x + 2\cos x) + C$$

$$I = \frac{e^{2x}}{5}(\sin x + 2\cos x) + C_1$$

Tabular Method

Can be useful when integral contains x^n.

Ex: $\int x^2 \cos 3x \, dx$

	D	I
+	x^2	$\cos 3x$
−	$2x$	$\frac{1}{3}\sin 3x$
+	2	$-\frac{1}{9}\cos 3x$
	0	$-\frac{1}{27}\sin 3x$

$$\int x^2 \cos 3x \, dx = \frac{x^2}{3}\sin 3x$$

$$+\frac{2x}{9}\cos 3x - \frac{2}{27}\sin 3x + C$$

Ex: $\int (x^3+2x)e^{2x}\,dx$

	D	I
+	(x^3+2x)	e^{2x}
−	$(3x^2+2)$	$e^{2x}/2$
+	$6x$	$e^{2x}/4$
−	6	$e^{2x}/8$
	0	$e^{2x}/16$

$$\int (x^3+2x)e^{2x}\,dx = \frac{(x^3+2x)e^{2x}}{2}$$

$$-\frac{(3x^2+2)e^{2x}}{4} + \frac{6xe^{2x}}{8}$$

$$-\frac{6e^{2x}}{16} + C$$

Ex: $\int 2x^3 \cos x^2 \, dx$

$$\boxed{\begin{aligned} u &= x^2 \\ du &= 2x \, dx \end{aligned}}$$

$$= \int 2x \cdot x^2 \cdot \cos x^2 \, dx$$

$$= \int u \cos u \, du$$

	D	I
+	u	$\cos u$
−	1	$\sin u$
	0	$-\cos u$

$$= u \sin u + \cos u + C$$

$$= x^2 \sin x^2 + \cos x^2 + C$$

8.3 Trig Integrals

We'll use substitution to find:

$$\int \sin^n\theta \cos\theta \, d\theta$$

OR

$$\int \cos^n\theta \sin\theta \, d\theta$$

Ex: $\int \sin^4\theta \cos\theta \, d\theta$

$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta \, d\theta \end{aligned}}$$

$$= \int u^4 \, du$$

$$= \frac{u^5}{5} + C$$

$$= \frac{1}{5}(\sin\theta)^5 + C$$

$$= \frac{1}{5}\sin^5\theta + C$$

Fact $\sin^2\theta + \cos^2\theta = 1$

<u>Ex:</u> $\int \sin^4\theta \cos^3\theta \, d\theta$

$$= \int \sin^4\theta \cos^2\theta \underbrace{\cos\theta \, d\theta}_{du}$$

$$= \int \sin^4\theta (1 - \sin^2\theta) \cos\theta \, d\theta$$

$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta \, d\theta \end{aligned}}$$

$$= \int u^4(1-u^2)\,du$$

$$= \int (u^4 - u^6)\,du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C$$

$$= \frac{1}{5}\sin^5\theta - \frac{1}{7}\sin^7\theta + C$$

<u>Ex</u>: $\int \sin^5\theta\,d\theta$

$$= \int \sin^4\theta \sin\theta\,d\theta$$

$$= \int (\sin^2\theta)^2 \sin\theta\,d\theta$$

$$= \int (1-\cos^2\theta)^2 \sin\theta\,d\theta$$

$$\boxed{\begin{aligned} u &= \cos\theta \\ du &= -\sin\theta\, d\theta \\ -du &= \sin\theta\, d\theta \end{aligned}}$$

$$= -\int (1-u^2)^2 \, du$$

$$= -\int (1-2u^2+u^4)\, du$$

$$= -\left[u - \frac{2}{3}u^3 + \frac{u^5}{5}\right] + C$$

$$= -\left[\cos\theta - \frac{2}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta\right] + C$$