Test 1
FRI SEPT 29
  7 Questions
Bring calculator
Bring music /earplugs
Practice Problems on website

Covers:

1.2-1.5	Limits
2.2-2.4	Polynomial/Trig Derivatives
5.1	Exp/Log Derivatives
2.5	Implicit Differentiation
4.4-4.5	Polynomial Integrals
5.2, 5.4	Exp/Log Integrals
5.7	Inverse Trig Derivatives
5.8	" Integrals
8.1	Basic Trig Integrals

5.8 Inverse Trig : Integration Cont'd

Ex: $\int \dfrac{2x+8}{x^2+2x+10} \, dx$

$$x^2 + 2x + 10$$
$$= x^2 + 2x + 1 + 9$$
$$= (x+1)^2 + 3^2$$

$$= \int \frac{2x+8}{3^2 + (x+1)^2}\, dx$$

$u = x+1$

$du = dx$

$2x + 8 = ?$

$2x + 8 = ?(x+1) + ?$

$\qquad = 2(x+1) + ?$

$\qquad = 2(x+1) + 6$

$\qquad = 2u + 6$

OR $\quad u - 1 = x$

$2x + 8 = 2(u-1) + 8$

$\qquad = 2u + 6$

$$= \int \frac{2u+6}{3^2 + u^2}\, du$$

$$= \int \left[\frac{2u}{3^2 + u^2} + \frac{6}{3^2 + u^2} \right] du$$

$$= \ln|3^2 + u^2| + \frac{6}{3} \tan^{-1} \frac{u}{3} + C$$

$$= \ln|3^2 + (x+1)^2| + 2\tan^{-1} \frac{x+1}{3} + C$$

5.8 #17

$$\int \frac{1}{\sqrt{x}\sqrt{1-x}}\, dx$$

$$= \int \frac{dx}{\sqrt{x}\sqrt{1-\sqrt{x}^2}}$$

$$= 2\int \frac{du}{\sqrt{1-u^2}}$$

$$u = \sqrt{x}$$
$$du = \frac{1}{2}x^{-1/2}\, dx$$
$$2du = \frac{dx}{\sqrt{x}}$$

$$= 2\sin^{-1}u + C$$

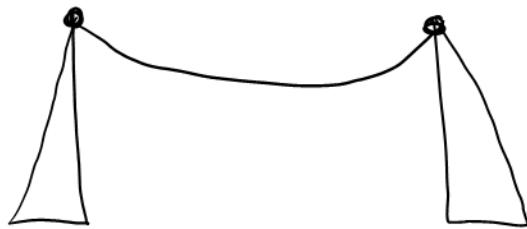$$= 2\sin^{-1}\sqrt{x} + C$$

5.9   Won't Be Tested
Hyperbolic Functions

Hyperbolic Cosine $\cosh x = \dfrac{e^x + e^{-x}}{2}$

Hyperbolic Sine $\sinh x = \dfrac{e^x - e^{-x}}{2}$

Quick Ex: Hanging Cable

$$y = a \cosh \frac{x}{a}, \quad a > 0$$

The shape is called a **catenary**.

$(\cosh t, \sinh t)$ lies on the hyperbola $x^2 - y^2 = 1$.

$$\frac{d}{dx}[\cosh x] = \sinh x$$

$$\frac{d}{dx}[\sinh x] = \cosh x$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

Ex: Find y'

a) $y = \sinh x^3$

$y' = 3x^2 \cosh x^3$

b) $y = x^2 \cosh 5x$

$y' = x^2 [\sinh 5x (5)] + (\cosh 5x)(2x)$

$= 5x^2 \sinh 5x + 2x \cosh 5x$

Ex: Find

a) $\int \cosh 4x \, dx$

$\begin{vmatrix} u = 4x \\ du = 4dx \\ \dfrac{du}{4} = dx \end{vmatrix}$

$= \frac{1}{4} \int \cosh u \, du$

$= \frac{1}{4} \sinh u + C$

$= \frac{1}{4} \sinh 4x + C$

b) $\int x^6 \sinh x^7 \, dx$

$= \frac{1}{7} \cosh x^7 + C$

8.1 Basic Trig Integrals

10 trig integrals on formula sheet.