$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}$$

<u>Ex</u>: Find y'

c) $y = 2x \arctan x - \ln(1+x^2)$

$$y' = 2x \frac{1}{1+x^2} + (\arctan x)(2) - \frac{1}{1+x^2}(2x)$$

$$= 2 \arctan x$$

d) $y = 12 \arcsin \frac{x}{4}$

$$y' = 12 \frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}} \cdot \frac{1}{4}$$

$$= \frac{12}{\sqrt{1-\frac{x^2}{16}}\ \sqrt{16}}$$

$$= \frac{12}{\sqrt{16-x^2}}$$

Ex: Prove that $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

$$\sin(\arcsin x) = x$$

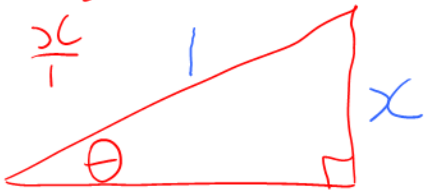
Take $\frac{d}{dx}$: $\cos(\arcsin x) \frac{d}{dx} \arcsin x = 1$

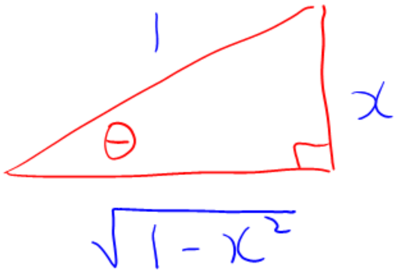
$$\frac{d}{dx} \arcsin x = \frac{1}{\cos(\arcsin x)}$$

Let $\theta = \arcsin x$

$\sin\theta = x$

$\sin\theta = \frac{x}{1}$





$$\cos\theta = \sqrt{1-x^2}$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\cos(\arcsin x)}$$

$$= \frac{1}{\cos\theta}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

5.8 Inverse Trig Functions: Integration

Let $a > 0$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \arcsin \frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a} \arctan \frac{x}{a} + C$$

<u>Ex</u>: Find $\int \frac{1}{\sqrt{16 - x^2}}\, dx$

$$= \arcsin \frac{x}{4} + C$$

<u>Ex</u>: Find $\int \frac{1}{9 + 4x^2}\, dx$

$$= \int \frac{dx}{3^2 + (2x)^2}$$

$$\boxed{\begin{aligned} u &= 2x \\ du &= 2\,dx \\ \frac{du}{2} &= dx \end{aligned}}$$

$$= \frac{1}{2} \int \frac{du}{3^2 + u^2}$$

$$= \frac{1}{2} \left(\frac{1}{3} \tan^{-1} \frac{u}{3} \right) + C$$

$= \frac{1}{6} \tan^{-1} \frac{2x}{3} + C$

<u>Ex</u>: Find $\displaystyle\int_0^{\pi/2} \frac{\cos\theta}{1+\sin^2\theta}\, d\theta$

$$\boxed{\begin{array}{l} u = \sin\theta \\ du = \cos\theta\, d\theta \\ \theta = 0 \Rightarrow u = 0 \\ \theta = \frac{\pi}{2} \Rightarrow u = 1 \end{array}}$$

$$= \int_0^1 \frac{du}{1+u^2}$$

$$= \arctan u \Big|_0^1$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$

Complete the Square

$x^2 + 10x + 34$

$$\boxed{\begin{array}{l} \frac{10}{2} = 5 \\ 5^2 = 25 \end{array}}$$

$$= x^2 + 10x + 25 + (34 - 25) \quad 9$$

$$= (x+5)^2 + 3^2$$

Ex: $\int \frac{dx}{x^2+6x+13}$

$$\frac{6}{2} = 3$$
$$3^2 = 9$$
$$x^2+6x+13$$
$$= x^2+6x+9 + \cancel{13-9}^{\,4}$$
$$= (x+3)^2+2^2$$

$$= \int \frac{dx}{(x+3)^2+2^2}$$

$$u = x+3$$
$$du = dx$$

$$= \int \frac{du}{2^2+u^2}$$

$$= \frac{1}{2}\tan^{-1}\frac{u}{2} + C$$

$$= \frac{1}{2}\tan^{-1}\frac{x+3}{2} + C$$

Ex: $\int \frac{dx}{\sqrt{8x-x^2}}$

$$8x-x^2$$
$$= -(x^2-8x)$$

$$\frac{-8}{2} = -4$$
$$(-4)^2 = 16$$

$$= -(x^2-8x+16)+16$$

$$= 4^2-(x-4)^2$$

$$= \int \frac{dx}{\sqrt{4^2-(x-4)^2}}$$

$$u = x-4$$
$$du = dx$$

$$= \int \frac{du}{\sqrt{4^2-u^2}}$$

$$= \arcsin \frac{u}{4} + C$$

$$= \arcsin \frac{x-4}{4} + C$$