2.5 Cont'd

Ex: y depends on x

a) $\frac{d}{dx}[x^3] = 3x^2$

b) $\frac{d}{dx}[y^3] = 3y^2\frac{dy}{dx}$

c) $\frac{d}{dx}[(7x^2)y^4]$

$= 7x^2\left(4y^3\frac{dy}{dx}\right) + y^4(14x)$

$= 28x^2y^3\frac{dy}{dx} + 14xy^4$

Ex: Find the tangent line to

$x^3+y^3 = 9xy$ at $(2,4)$.

$(9x)y$

1) Take $\frac{d}{dx}$

$3x^2 + 3y^2\frac{dy}{dx} = 9x\frac{dy}{dx} + y(9)$

2) Solve for $\frac{dy}{dx}$

$3y^2\frac{dy}{dx} - 9x\frac{dy}{dx} = 9y - 3x^2$

$$\left[3y^2 - 9x\right]\frac{dy}{dx} = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x}$$

$$\left.\frac{dy}{dx}\right|_{(2,4)} = \frac{24}{30} = \frac{4}{5}$$

$$m = \frac{4}{5} \qquad x_1 = 2 \qquad y_1 = 4$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{4}{5}(x - 2)$$

$$y - 4 = \frac{4}{5}x - \frac{8}{5}$$

$$y = \frac{4}{5}x + \frac{12}{5}$$

Ex: Find $\frac{dy}{dx}$:

$$x^4 y + y^4 = 1 + \sin(xy)$$

Take $\frac{d}{dx}$:

$$x^4\frac{dy}{dx} + y(4x^3) + 4y^3\frac{dy}{dx} = \cos(xy)\left[x\frac{dy}{dx} + y(1)\right]$$

Solve for $\frac{dy}{dx}$:

$$x^4\frac{dy}{dx} + 4x^3y + 4y^3\frac{dy}{dx} = x\cos(xy)\frac{dy}{dx} + y\cos(xy)$$

$$x^4\frac{dy}{dx} + 4y^3\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy) - 4x^3y$$

$$\left[x^4 + 4y^3 - x\cos(xy)\right]\frac{dy}{dx} = y\cos(xy) - 4x^3y$$

$$\frac{dy}{dx} = \frac{y\cos(xy) - 4x^3y}{x^4 + 4y^3 - x\cos(xy)}$$

4.4-4.5 Review of Integration

The indefinite integral $\int f(x)\,dx$ represents all antiderivatives of $f(x)$.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

Ex: Find

a) $\int (x^5 + 2x^4 - 5x + 3)\,dx$

$$= \frac{x^6}{6} + \frac{2x^5}{5} - \frac{5x^2}{2} + 3x + C$$

b) $\int \left(\sqrt{x} + \frac{1}{x^3}\right) dx$

$$= \int \left(x^{1/2} + x^{-3}\right) dx$$

$$= \frac{2}{3}x^{3/2} - \frac{1}{2}x^{-2} + C$$

or $\frac{2x^{3/2}}{3} - \frac{1}{2x^2} + C$

Ex: Find

a) $\int 5x^2 (x^3+1)^6 \, dx$

$u = x^3 + 1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$$= \int \frac{5}{3} u^6 \, du$$

$$= \frac{5}{3} \int u^6 \, du$$

$$= \frac{5}{3}\frac{u^7}{7} + C$$

$$= \frac{5}{21}\left(x^3+1\right)^7 + C$$

b) $$\int \frac{x}{\sqrt{2x^2+1}}\,dx$$

$$u = 2x^2+1$$
$$\frac{du}{dx} = 4x$$
$$du = 4x\,dx$$
$$\frac{du}{4} = x\,dx$$

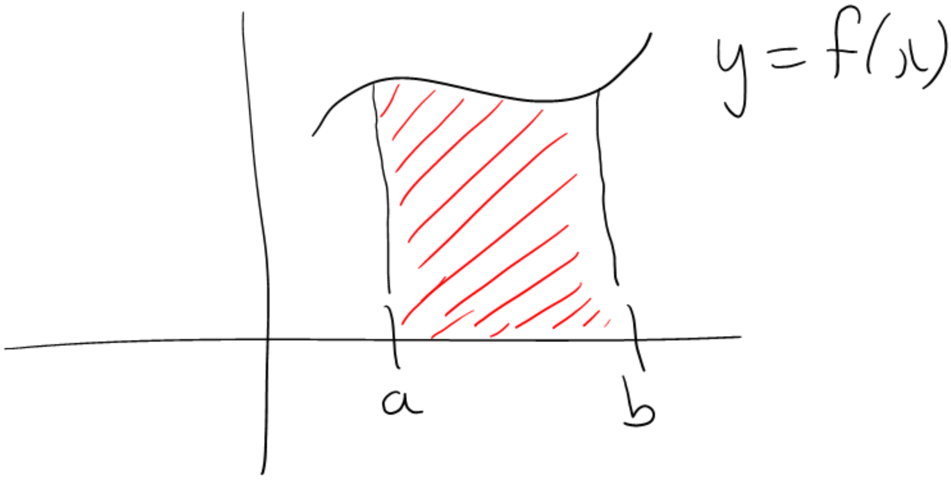
$$= \frac{1}{4}\int \frac{du}{\sqrt{u}}$$

$$= \frac{1}{4}\int u^{-1/2}\,du$$

$$= \frac{1}{4}\left(2u^{1/2}\right) + C$$

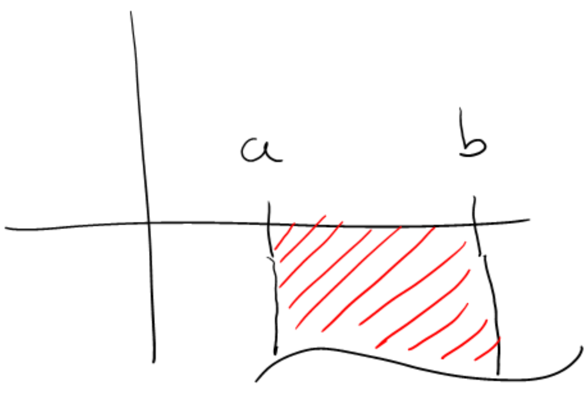
$$= \frac{1}{2}\sqrt{2x^2+1} + C$$

The definite integral $\int_a^b f(x)\,dx$ represents area under a curve, if $f(x) \geqslant 0$.



$$\int_a^b f(x)\,dx < 0$$

when the curve is below the x-axis.



$$\int_a^b f(x)\,dx = \mathbf{F}(b) - \mathbf{F}(a)$$

where $\mathbf{F}(x)$ is an antiderivative for $f(x)$.

$$\int_1^4 x^2\,dx = \frac{x^3}{3}\Bigg|_1^4$$

$$= \frac{64}{3} - \frac{1}{3}$$

$$= \frac{63}{3}$$

$$= 21$$

$f'(x) = 2x \qquad f(3) = 11$

Find $f(x)$

$$f(x) = x^2 + C$$

$f = 11$, $x = 3$; $\quad 11 = 9 + C$

$$C = 2$$

$$f(x) = x^2 + 2$$