(32) $\vec{v}(t) = \int \vec{a}(t)\,dt$

$= [4t, 0, -9.8t] + \vec{C}_1$

Sub $t=0$: $[0, 25, 25] = [0, 0, 0] + \vec{C}_1$

$\vec{C}_1 = [0, 25, 25]$

$$\boxed{\vec{v}(t) = [4t, 25, 25 - 9.8t]}$$

$\vec{r}(t) = \int \vec{v}(t)\,dt$

$= [2t^2, 25t, 25t - 4.9t^2] + \vec{C}_2$

Sub $t=0$: $\vec{0} = \vec{0} + \vec{C}_2$

$\vec{C}_2 = \vec{0}$

$$\boxed{\vec{r}(t) = [2t^2, 25t, 25t - 4.9t^2]}$$

It hits the ground when

(z-component of $\vec{r}(t)$) $= 0$

$25t - 4.9t^2 = 0$

$t(25 - 4.9t) = 0$

$t = 0$ or $25 - 4.9t = 0$

$t = \frac{25}{4.9}$

$\vec{v}\left(\frac{25}{4.9}\right) = \left[\frac{100}{4.9}, 25, -25\right]$

$$\| \vec{v}\left(\tfrac{25}{4.9}\right) \| = \sqrt{\left(\tfrac{100}{4.9}\right)^2 + 25^2 + (-25)^2}$$

$$\approx 40.8 \text{ m/s}$$

(33) $\vec{r} = [t, 1 + \frac{1}{t}, 0]$

$\vec{v} = [1, -t^{-2}, 0]$

$\vec{a} = [0, 2t^{-3}, 0]$

$$a_T = \frac{\vec{v} \cdot \vec{a}}{\|\vec{v}\|}$$

$$\vec{v} \cdot \vec{a} = 1(0) + (-t^{-2})(2t^{-3}) + 0(0)$$
$$= -2t^{-5}$$

$$\|\vec{v}\| = \sqrt{1^2 + (-t^{-2})^2 + 0^2}$$
$$= \sqrt{1 + t^{-4}}$$

$$a_T = \frac{\vec{v} \cdot \vec{a}}{\|\vec{v}\|}$$
$$= \frac{-2t^{-5}}{\sqrt{1+t^{-4}}}$$

$$a_N = \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|}$$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -t^{-2} & 0 \\ 0 & 2t^{-3} & 0 \end{vmatrix} = (0)\vec{i} - (0)\vec{j} + (2t^{-3})\vec{k}$$

$$\vec{v} \times \vec{a} = [0, 0, 2t^{-3}]$$

$$\|\vec{v} \times \vec{a}\| = \sqrt{0^2 + 0^2 + (2t^{-3})^2}$$

$$= \sqrt{4t^{-6}}$$

$$= |2t^{-3}|$$

$$= 2t^{-3} \qquad (t > 0)$$

$$a_N = \frac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}\|}$$

$$= \frac{2t^{-3}}{\sqrt{1 + t^{-4}}}$$

(34)

$$s = \int \|\vec{v}\| \, dt$$

$$\vec{v} = [3\cos^2 t(-\sin t), 3\sin^2 t \cos t]$$

$$\|\vec{v}\| = \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}$$

$$= \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}$$

$$= \sqrt{9\cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)}$$

$$= |3\cos t \sin t|$$

$$= 3\cos t \sin t \qquad (0 \le t \le \tfrac{\pi}{2})$$

$$= \frac{3}{2}\sin 2t$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$S = \int_0^{\frac{\pi}{2}} \frac{3}{2} \sin 2t \, dt$$

$$= -\frac{3}{4} \cos 2t \Big|_0^{\pi/2}$$

$$= \frac{3}{4} - \left(-\frac{3}{4}\right)$$

$$= \frac{3}{2}$$