Test 4 #3

Rewrite as a power series centred at $c = -1$:

$$f(x) = \frac{3}{7x+6}$$

$$= \frac{3}{7(x+1) + ?}$$

$$= \frac{3}{7(x+1) - 1}$$

$$= \frac{-3}{1 - 7(x+1)}$$

$$= -3 \sum_{n=0}^{\infty} \left[7(x+1) \right]^n$$

$$= -3 \sum_{n=0}^{\infty} 7^n (x+1)^n$$

ASIDE $f(x) = \frac{3}{7x+4}$

$$= \frac{3}{7(x+1) - 3}$$

$$= \frac{-3}{3 - 7(x+1)}$$

$$= \frac{-3}{3} \cdot \frac{1}{1 - \frac{7}{3}(x+1)}$$

$$= - \sum_{n=0}^{\infty} \left[\frac{7}{3}(x+1) \right]^n$$

$\sum x^n$ centred at $c=0$

$\sum (x-4)^n$ " $c=4$

$\sum (x+1)^n$ $c=-1$

ASIDE $f(x)=\frac{3}{7x+6}$ Power series centred at $c=-5$?

$$= \frac{3}{7(x+5)-29}$$

$$= \frac{-3}{29-7(x+5)}$$

$$= \frac{-3}{29}\cdot\frac{1}{1-\frac{7}{29}(x+5)}$$

$$= \frac{-3}{29}\sum_{n=0}^{\infty}\left[\frac{7}{29}(x+5)\right]^n$$

Review Problems on website

(24) $e^x \approx 1+x+\frac{x^2}{2}+\frac{x^3}{6}$

$$e^{-2x} \approx 1-2x+\frac{(-2x)^2}{2}+\frac{(-2x)^3}{6}$$

$$\approx 1-2x+2x^2-\frac{4x^3}{3}$$

$$x e^{-2x} \approx x-2x^2+2x^3-\frac{4x^4}{3}$$

(25) $(1+x)^{1/3} \approx 1 + \frac{1}{3}x + \frac{1}{3}\left(\frac{-2}{3}\right)\frac{x^2}{2}$

$\approx 1 + \frac{x}{3} - \frac{x^2}{9}$

$(1+x^2)^{1/3} \approx 1 + \frac{x^2}{3} - \frac{x^4}{9}$

$$\int_0^{0.5} \sqrt[3]{1+x^2}\,dx \approx \int_0^{0.5} \left(1 + \frac{x^2}{3} - \frac{x^4}{9}\right)dx$$

$$\approx x + \frac{x^3}{9} - \frac{x^5}{45} \Big|_0^{0.5}$$

$$\approx 0.5132$$

(21) g) Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(n+1)!}\ (n+1)}{1(3)(5)\cdots(2n+1)} \cdot \frac{1(3)(5)\cdots(2n-1)}{\cancel{n!}} \right|$$

$$2\cancel{n}-1$$
$$(n+1)$$
$$= 2n+2-1$$
$$= 2n+1$$

$$= \lim_{n\to\infty} \left| \frac{n+1}{2n+1} \right|$$

$$= \lim_{n\to\infty} \frac{n+1}{2n+1}$$

$$\overset{H}{=} \frac{1}{2}$$

Series Converges.

h) Root Test

$$\lim_{n\to\infty} \sqrt[n]{|a_n|}$$

$$= \lim_{n\to\infty} \sqrt[n]{\left|\frac{1}{(1+\ln n)^n}\right|}$$

$$= \lim_{n\to\infty} \sqrt[n]{\frac{1}{(1+\ln n)^n}}$$

$$= \lim_{n\to\infty} \frac{1}{1+\ln n}$$

$$= 0$$

Series converges.

22 $$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$$

$f(x) = e^x$ $\qquad x = -0,2$

$f'(x) = e^x$

$f''(x) = e^x$

$f^{(n+1)}(x) = e^x$

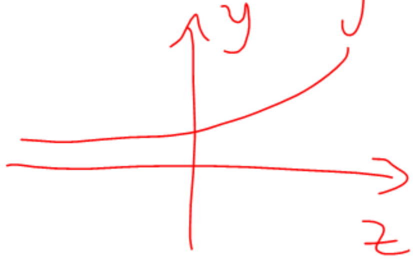
$f^{(n+1)}(z) = e^z$

$c = 0$ (Maclaurin)

z: between $\cancel{x}$ and $\cancel{c}$

$-0,2$ and 0

$e^z \le \cancel{e^0}\ 1$



$$|R_N(x)| = \left| \frac{e^z}{(N+1)!} (-0,2)^{N+1} \right|$$

$$\le \frac{1}{(N+1)!} (0.2)^{N+1}$$

$$\frac{0,2^{N+1}}{(N+1)!} < 0,0001$$

Guess and check

$N=1$? No

$N=2$? NO

$N=3$? YES

$$\boxed{N \ge 3}$$

23 Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-1)^{n+1}}{(n+1)^2\cdot 2^{n+1}}\cdot\frac{n^2\cdot 2^n}{(x-1)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-1)}{2}\cdot\frac{n^2}{(n+1)^2}\right|$$

$$\overset{H}{=} \lim_{n\to\infty}\left|\frac{x-1}{2}\quad\frac{2n}{2(n+1)}\right|$$

$$= \lim_{n \to \infty} \left| \frac{x-1}{2} \cdot \cancel{\frac{2}{2}} \right|$$

$$= \left| \frac{x-1}{2} \right|$$

Series converges if $\left| \frac{x-1}{2} \right| < 1$

$$|x-1| < 2$$

$$-2 < x-1 < 2$$

$$-1 < x < 3$$

Endpoints:

$x = -1$

$$\text{Series} = \sum_{n=1}^{\infty} \frac{(-2)^n}{n^2 \cdot 2^n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n \cancel{2^n}}{n^2 \cdot \cancel{2^n}}$$

$x = 3$

$$\text{Series} = \sum_{n=1}^{\infty} \frac{\cancel{2^n}\ 1}{n^2 \cdot \cancel{2^n}}$$

Converges

(p-series with $p = 2$)

Converges
by Alternating
Series Test

$$\boxed{-1 \leq x \leq 3}$$