Test Average: 74%

Review Problems are online.

FINAL EXAM

Thurs Dec 14

1:30 - 4:30 pm

TEC 175

No Music Allowed

Bring calculator, earplugs

14 Questions

Section	% of Marks on Exam
8.2 - 8.5, 5.6, 8.8	30
9.1 - 9.10	28
10.2 - 10.5	25
12.1 - 12.5	17

(16) $\displaystyle\int_0^\infty \frac{e^x}{1+(e^x)^2}\,dx$

$u = e^x$

$du = e^x\,dx$

$x = 0 \Rightarrow u = 1$

$x \to \infty \Rightarrow u \to \infty$

$$= \int_1^\infty \frac{du}{1+u^2}$$

$$= \lim_{b\to\infty} \int_1^b \frac{du}{1+u^2}$$

$$= \lim_{b\to\infty} \arctan u \Big|_1^b$$

$$= \lim_{b\to\infty} \arctan b - \arctan 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4}$$

18 a)

$$\frac{7}{(n+3)(n+4)} = \frac{A}{n+3} + \frac{B}{n+4}$$

$$7 = A(n+4) + B(n+3)$$

$n=-4$: $\quad 7 = -B \implies B = -7$

$n=-3$: $\quad 7 = A$

$$\text{Series} = \sum_{n=2}^{\infty} \left(\frac{7}{n+3} - \frac{7}{n+4} \right)$$

$$= \left(\frac{7}{5} - \cancel{\frac{7}{6}} \right) + \left(\cancel{\frac{7}{6}} - \cancel{\frac{7}{7}} \right) + \cdots$$

$$= \frac{7}{5} - \lim_{n\to\infty} \frac{7}{n+4}$$

$$= \frac{7}{5}$$

b)

$$\sum_{n=2}^{\infty} \frac{2^{n+1}}{7^n}$$

$$= \frac{8}{49} + \frac{16}{343} + \cdots$$

$\times r \qquad \times r$

$$\boxed{\text{Geometric} \quad a = \frac{8}{49} \qquad r = \frac{2}{7}}$$

$$= \frac{a}{1-r}$$

$$= \frac{\left(\frac{8}{49}\right)}{\left(\frac{5}{7}\right)}$$

$$= \frac{8}{49} \cdot \frac{7}{5}$$

$$= \frac{8}{35}$$

19 Compute $\int_N^\infty \frac{2}{x^2}\,dx$.

Then $\int_N^\infty \frac{2}{x^2}\,dx \leq 0.1$

$$\int_N^\infty 2x^{-2}\,dx = \lim_{b\to\infty} \int_N^b 2x^{-2}\,dx$$

$$= \lim_{b\to\infty} -2x^{-1} \Big|_N^b$$

$$= \lim_{b\to\infty} -\frac{2}{b} + \frac{2}{N}$$

$$= \frac{2}{N}$$

$$\frac{2}{N} \leq 0.1$$

$$2 \le 0.1N$$

$$20 \le N$$

$$N \geq 20$$

20 a) $S_3 = -1 + \frac{1}{4} - \frac{1}{9}$

$$= \frac{-31}{36}$$

b) $|R_3| \le a_4$ unsigned

$$\le \frac{1}{16}$$

c) $\frac{-31}{36} - \frac{1}{16} \le \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \le \frac{-31}{36} + \frac{1}{16}$

21 a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$

Converges

(p-series with $p = 1.1$)

b) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

ASIDE

$$\lim_{n\to\infty} n^2 e^{-n^3} = \lim_{n\to\infty} \frac{n^2}{e^{n^3}}$$

$$\overset{H}{=} 0$$

$$f(x) = x^2 e^{-x^3}$$

$f(x)$ is continuous

$f(x) > 0$ on $[1, \infty)$

$$f'(x) = x^2\left(-3x^2 e^{-x^3}\right) + 2x e^{-x^3}$$
$$= \left(-3x^4 + 2x\right) e^{-x^3}$$
$$< 0 \quad \text{on} \quad [1, \infty)$$

Use the Integral Test.

$$\int_1^\infty x^2 e^{-x^3}\,dx$$

$$\boxed{\begin{aligned} u &= -x^3 \\ du &= -3x^2\,dx \\ -\tfrac{1}{3}\,du &= x^2\,dx \\ x = 1 &\Rightarrow u = -1 \\ x \to \infty &\Rightarrow u \to -\infty \end{aligned}}$$

$$= -\frac{1}{3}\int_{-1}^{-\infty} e^u\,du$$

$$= \lim_{b \to -\infty} -\frac{1}{3}\int_{-1}^{b} e^u\,du$$

$$= \lim_{b \to -\infty} -\frac{1}{3} e^u \Big|_{-1}^{b}$$

$$= \lim_{b \to -\infty} -\frac{1}{3} e^b + \frac{1}{3} e^{-1}$$

$= \frac{1}{3}e^{-1}$

Series Converges by the Integral Test.

c) Series diverges by n^{th} Term Test:

$$\lim_{n\to\infty} \frac{n}{2n+1} \overset{H}{=} \frac{1}{2}$$

d) Alternating Series ✓

$$\lim_{n\to\infty} \frac{1}{2n} = 0 \quad ✓$$

$a_{n+1} \leq a_n$? $\quad \frac{1}{2(n+1)} \leq \frac{1}{2n}$ ✓

Series Converges by Alternating Series Test.

e) Direct Comparison

$a_n = \frac{1}{3^{(n^3)}} \qquad b_n = \frac{1}{3^n}$

$$0 < \frac{1}{3^{(n^3)}} \leq \frac{1}{3^n} \quad ✓$$

and $\sum_{n=1}^{\infty} \frac{1}{3^n}$ Converges (geometric $r = \frac{1}{3}$)

Therefore $\sum_{n=1}^{\infty} \frac{1}{3^{(n^3)}}$ Converges.

f) Limit Comparison

$$a_n = \frac{\sqrt{n}+3}{7n^2+2}$$

Dominant terms : $\frac{\sqrt{n}}{n^2} = n^{-3/2}$

$$b_n = n^{-3/2}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\frac{\sqrt{n}+3}{7n^2+2}\right)(n^{3/2})$$

$$= \lim_{n\to\infty} \frac{n^2+3n^{3/2}}{7n^2+2}$$

$$= \lim_{n\to\infty} \frac{1+3n^{-1/2}}{7+2n^{-2}}$$

$$= \frac{1}{7}$$

$$0 < L < \infty \checkmark$$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ Converges (p-series with $p = 3/2$)

$\Rightarrow$ The series Converges.