(11)

$u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^3}{3}$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \ln x\,dx = \frac{x^3}{3}\ln x - \int \frac{1}{3} x^2\,dx$

$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

(12) $\int \sec^4\theta \tan^3\theta\,d\theta$

$= \int \underbrace{\sec^2\theta}_{(1+\tan^2\theta)} (\tan^3\theta) \underbrace{\sec^2\theta\,d\theta}_{du}$

$= \int (\tan^3\theta + \tan^5\theta)\sec^2\theta\,d\theta$

$\boxed{\begin{aligned} u &= \tan\theta \\ du &= \sec^2\theta\,d\theta \end{aligned}}$

$$= \int (u^3 + u^5)\,du$$

$$= \frac{u^4}{4} + \frac{u^6}{6} + C$$

$$= \frac{\tan^4\theta}{4} + \frac{\tan^6\theta}{6} + C$$

13

$\sqrt{a^2+x^2}$

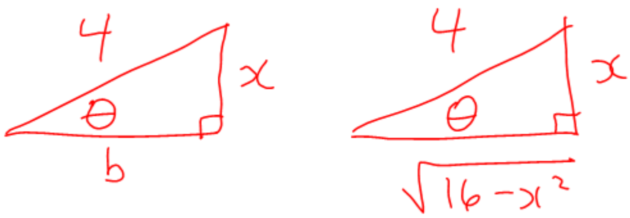
$\sqrt{a^2-x^2}$ $\longrightarrow$ TRIG SUB

$\sqrt{x^2-a^2}$

Sub $x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

$$\frac{x}{4} = \sin\theta$$




$\left\{ \begin{array}{l} 4^2 - x^2 = b^2 \\ \vdots \end{array} \right.$

$$\frac{\sqrt{16-x^2}}{4} = \cos\theta$$

$$\sqrt{16-x^2} = 4\cos\theta$$

$$\text{Integral} = \int \frac{dx}{x^2\sqrt{16-x^2}}$$

$$= \int \frac{\cancel{4\cos\theta}\, d\theta}{(4\sin\theta)^2\,\cancel{(4\cos\theta)}}$$

$$= \frac{1}{16}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{16}\cot\theta + C$$

$$= \frac{-1}{16}\,\frac{\sqrt{16-x^2}}{x} + C$$

14

$$\frac{13}{(x+2)(x^2+9)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+9}$$

$$13 = A(x^2+9) + (Bx+C)(x+2)$$

$x=-2$: $\quad 13 = 13A \Rightarrow A = 1$

$x^2$ Coefficient: $\quad 0 = A + B \Rightarrow B = -1$

$x=0$: $\quad 13 = 9A + C(2)$

$$13 = 9 + 2C$$

$$C = 2$$

$$\text{Integral} = \int \left[ \frac{1}{x+2} + \frac{-x+2}{x^2+9} \right] dx$$

$$= \int \left[ \frac{1}{x+2} - \frac{x}{x^2+9} + \frac{2}{x^2+9} \right] dx$$

$$= \ln|x+2| - \frac{1}{2}\ln|x^2+9| + \frac{2}{3}\tan^{-1}\frac{x}{3} + C$$

15

a) $\displaystyle \lim_{x\to 0} \frac{\sin 6x}{\tan 7x} \leftarrow \frac{0}{0}$

$\overset{H}{=} \lim_{x \to 0} \dfrac{6\cos 6x}{7\sec^2 7x}$

$= \dfrac{6}{7}$

b) $\lim_{x \to 0^+} (e^x + 5x)^{\frac{1}{x}} \leftarrow 1^{\infty}$

Let $L = \lim_{x \to 0^+} (e^x + 5x)^{\frac{1}{x}}$

$\ln L = \lim_{x \to 0^+} \cancel{\ln(e^x + 5x)^{1/x}}$

$\dfrac{\ln(e^x + 5x)}{x} \leftarrow \dfrac{0}{0}$

$\overset{H}{=} \lim_{x \to 0^+} \dfrac{\left(\frac{1}{e^x + 5x}\right)(e^x + 5)}{1}$

$= \dfrac{\left(\frac{1}{1}\right)(6)}{1}$

$= 6$

$\ln L = 6 \qquad L = e^6$