Test Review

Ex: Let $\vec{r}(t) = [7t - \cos 2t, 3t^2, e^{-t}]$

Find $\vec{r}\,'(0) \cdot \vec{r}\,''(0)$ and $\vec{r}\,'(0) \times \vec{r}\,''(0)$.

$\vec{r}\,'(t) = [7 + 2\sin 2t, 6t, -e^{-t}]$

$\vec{r}\,''(t) = [4\cos 2t, 6, e^{-t}]$

$\vec{r}\,'(0) = [7, 0, -1]$

$\vec{r}\,''(0) = [4, 6, 1]$

$\vec{r}\,'(0) \cdot \vec{r}\,''(0) = 28 + 0 - 1$
$= 27$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 0 & -1 \\ 4 & 6 & 1 \end{vmatrix}$$

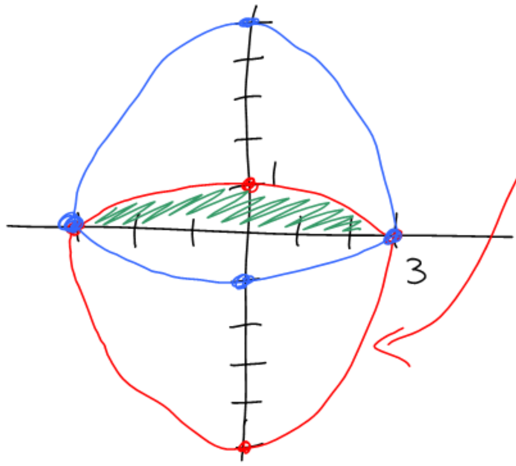
$\vec{r}\,'(0) \times \vec{r}\,''(0) = \vec{i}(6) - \vec{j}(11) + \vec{k}(42)$
$= [6, -11, 42]$

Exam Review Problems

Full Solutions on Website

Questions 1-10 are review material from beginning of course.

30



$r = 3 - 2\sin\theta$

$r = -3 + 2\sin\theta$

A = double the shaded area
(by symmetry)

$$= 2 \cdot \frac{1}{2} \int_0^{\pi} (3-2\sin\theta)^2 \, d\theta$$

$$= \int_0^{\pi} \left(9 - 12\sin\theta + \underbrace{\cancel{4\sin^2\theta}}_{2-2\cos 2\theta}\right) d\theta$$

$$= \left[9\theta + 12\cos\theta + 2\theta - \sin 2\theta\right]_0^{\pi}$$

$$= (9\pi - 12 + 2\pi) - (12)$$

$$= 11\pi - 24$$

(29) $r = 1 + \sin\theta$

$$x = r\cos\theta$$

$$x = (1+\sin\theta)\cos\theta$$

$$\frac{dx}{d\theta} = (1+\sin\theta)(-\sin\theta) + \cos^2\theta$$

Set $\frac{dx}{d\theta} = 0$: $\quad 0 = -\sin\theta - \sin^2\theta + \underbrace{\cancel{\cos^2\theta}}_{1-\sin^2\theta}$

$$0 = -2\sin^2\theta - \sin\theta + 1$$

$$0 = (-2\sin\theta + 1)(\sin\theta + 1)$$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\sin\theta = -1$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$y = r\sin\theta$$

$$y = (1+\sin\theta)\sin\theta \qquad y = \sin\theta + \sin^2\theta$$

$$\frac{dy}{d\theta} = \cos\theta + 2\sin\theta\cos\theta$$

Set $\frac{dy}{d\theta} = 0$: $\quad 0 = \cos\theta + 2\sin\theta\cos\theta$

$$0 = \cos\theta(1 + 2\sin\theta)$$

$\cos\theta = 0$ $\qquad$ $\sin\theta = \frac{-1}{2}$

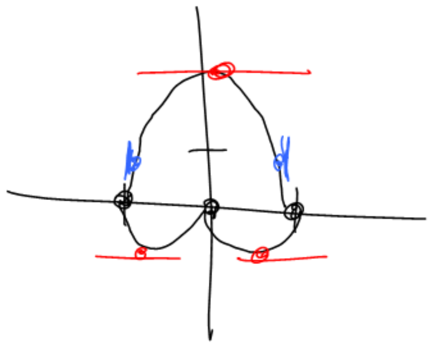
$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Hor. Tangent $\Rightarrow$ $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} \neq 0$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Vert. Tangent $\Rightarrow \frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$r = 1 + \sin\theta$

(28) $x = \cos t \qquad y = t - \sin t$

$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = 1 - \cos t$

$$S = \int_0^{\pi} \sqrt{\sin^2 t + 1 - 2\cos t + \cos^2 t}\, dt$$

$$= \int_0^{\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{\pi} \sqrt{2(1 - \cos t)}\, dt$$

$$\sin^2\theta = \frac{1}{2} - \frac{\cos 2\theta}{2}$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$$2\sin^2\frac{t}{2} = 1 - \cos t$$

$$= \int_0^{\pi} \sqrt{2\left(2\sin^2\frac{t}{2}\right)}\,dt$$

$$= \int_0^{\pi} \left|2\sin\frac{t}{2}\right| dt$$

$$= \int_0^{\pi} 2\sin\frac{t}{2}\,dt$$

$$= -4\cos\frac{t}{2}\Big|_0^{\pi}$$

$$= 0 - (-4)$$

$$= 4$$

31 a) $\frac{d}{dt} 9t\,[t^2, t^3]$

$= \frac{d}{dt} [9t^3, 9t^4]$

$= [27t^2, 36t^3]$

$\frac{d}{dt} 9t\,[t^2, t^3]$

$= 9t\,[2t, 3t^2] + 9\,[t^2, t^3]$

$= [18t^2, 27t^3] + [9t^2, 9t^3]$

$= [27t^2, 36t^3]$ ✓

b) $\frac{d}{dt} \vec{r}(2t)$

$= \frac{d}{dt} [14t+1, 8t]$

$= [14, 8]$

$$\frac{d}{dt}\vec{r}(2t)$$

$$= \vec{r}\,'(2t)\,(2)$$

$$= [7,4]\,(2)$$

$$\boxed{\vec{r}\,'(t) = [7,4]}$$

$$= [14, 8]$$

d) $$\int_1^3 [6t^2, 8t]\,dt$$

$$= [2t^3, 4t^2]\Big|_1^3$$

$$= [54, 36] - [2, 4]$$

$$= [52, 32]$$

ASIDE

$$\int [6t^2, 8t]\,dt$$

$$= [2t^3 + C_1, 4t^2 + C_2]$$

$$= [2t^3, 4t^2] + \vec{C}$$