# FINAL EXAM

Thurs Dec 14

1:30 – 4:30 pm

TEC 175

No music allowed

Bring: calculator, earplugs

**14 Questions**

| Sections | % of Marks on Exam |
|---|---|
| 8.2-8.5, 5.6, 8.8 | 30 |
| 9.1-9.10 | 28 |
| 10.2-10.5 | 25 |
| 12.1-12.5 | 17 |

## 12.3 Velocity and Acceleration Cont'd

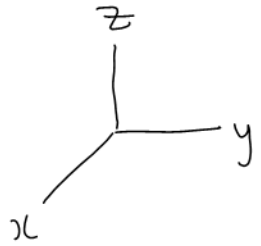
Ex: A ball is thrown with spin such that

$\vec{a} = [0.60, 0, -9.8]$,

$\vec{v}(0) = [0, 10, 10]$,

and $\vec{r}(0) = [0, 0, 0]$.

Find where, and with what speed, the ball hits the ground.



$$\vec{v} = \int \vec{a}\, dt$$

$$= [0.60t, 0, -9.8t] + \vec{C}_1$$

Sub $t=0$: $[0, 10, 10] = [0, 0, 0] + \vec{C}_1$

$$\vec{C}_1 = [0, 10, 10]$$

$$\vec{v} = [0.60t, 10, 10 - 9.8t]$$

$$\vec{r} = \int \vec{v}\, dt$$

$$= [0.30t^2, 10t, 10t - 4.9t^2] + \vec{C}_2$$

Sub $t=0$: $[0, 0, 0] = [0, 0, 0] + \vec{C}_2$

$$\vec{C}_2 = [0, 0, 0]$$

$$\vec{r} = [0.30t^2, 10t, 10t - 4.9t^2]$$

Time when ball hits the ground:

($z$-component of $\vec{r}$) $= 0$

$$10t - 4.9t^2 = 0$$

$$t(10 - 4.9t) = 0$$

$$t = 0, \quad t = \frac{10}{4.9}$$

Position when ball hits the ground:

$$\vec{r}\left(\frac{10}{4.9}\right) = [\ 1.2\ ,\ 20\ ,\ 0\ ]$$

$$\|\vec{v}(t)\| = \sqrt{(0.60t)^2 + 100 + (10 - 9.8t)^2}$$

Speed when ball lands:

$$\|\vec{v}\left(\frac{10}{4.9}\right)\| \approx 14 \text{ m/s}$$

## 12.4 Tangent and Normal Vectors

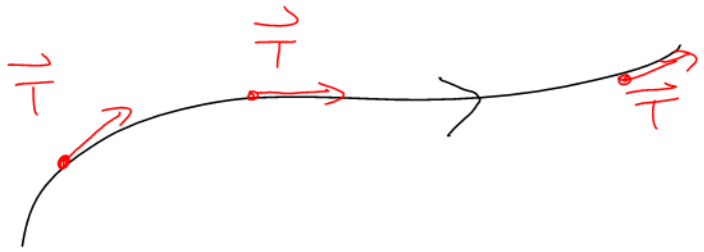
A curve $\vec{r}(t)$ is <u>smooth</u> on $a \le t \le b$ if $\vec{r}'(t)$ is continuous and nonzero on $a \le t \le b$.

<u>Def</u>

A <u>unit tangent vector</u> to a smooth curve $\vec{r}(t)$ is:

$$\vec{T}(t) = \frac{\vec{v}(t)}{\|\vec{v}(t)\|}$$

It's tangent to $\vec{r}(t)$ and it has $\|\vec{T}(t)\| = 1$.




Ex: Show that $\vec{T}'(t) \perp \vec{T}(t)$ for all $t$.

$$\|\vec{T}(t)\| = 1$$

$$\|\vec{T}(t)\|^2 = 1$$

$$\|\vec{w}\|^2 = \vec{w} \cdot \vec{w}$$

$$\vec{T}(t) \cdot \vec{T}(t) = 1$$

Take $\frac{d}{dt}$: $\vec{T}(t) \cdot \vec{T}'(t) + \vec{T}'(t) \cdot \vec{T}(t) = 0$

$$2\left[\vec{T}'(t) \cdot \vec{T}(t)\right] = 0$$

$$\vec{T}'(t) \cdot \vec{T}(t) = 0$$

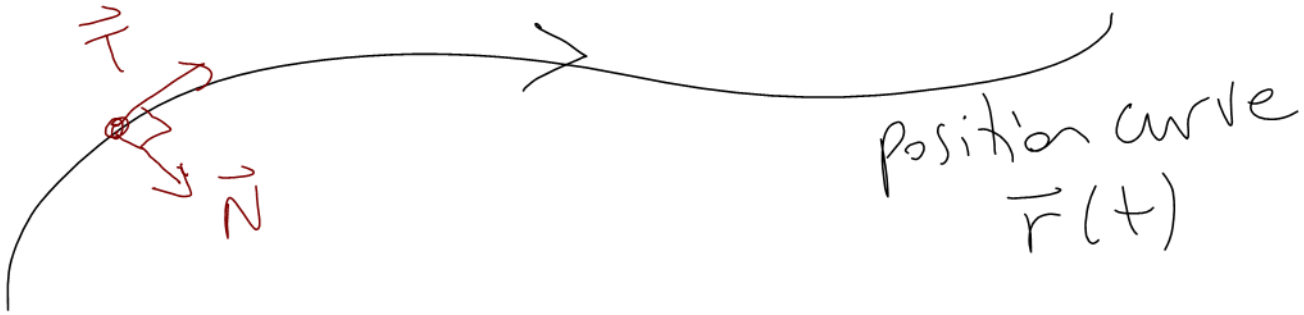
$$\vec{T}'(t) \perp \vec{T}(t)$$

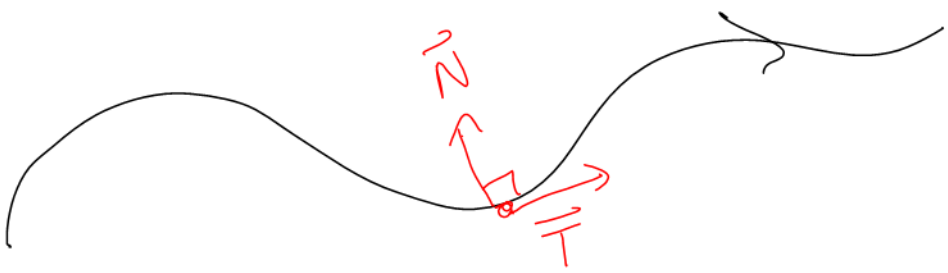
Def

A <u>unit normal vector</u> to a smooth curve $\vec{r}(t)$ is:

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$$

It's perpendicular to $\vec{r}(t)$ (pointing towards the concave side) and has length 1.



FACT

$$\vec{a}(t) = a_T(t)\,\vec{T}(t) + a_N(t)\,\vec{N}(t)$$

functions of $t$

where $a_T(t) = \dfrac{\vec{v}\cdot\vec{a}}{\|\vec{v}\|}$

"tangential component of acceleration"

and

$$a_N(t) = \frac{\|\vec{v}\times\vec{a}\|}{\|\vec{v}\|}$$

"normal component of acceleration"