(2)

$$0 < \frac{0.4 + 0.5|\sin n|}{n^{1.5}} \le \frac{1}{n^{1.5}} \quad \text{for all } n \geq 1 \checkmark$$

and $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$ Converges (p-series)

$$\Rightarrow \sum_{n=1}^{\infty} \frac{0.4 + 0.5|\sin n|}{n^{1.5}} \text{ Converges}$$

(6)

$$\int_N^{\infty} \frac{1}{x^{3.5}}\, dx$$

$$= \lim_{b \to \infty} \int_N^b x^{-3.5}\, dx$$

$$= \lim_{b \to \infty} \frac{1}{-2.5} x^{-2.5} \Big|_N^b$$

$$= \lim_{b \to \infty} \frac{1}{-2.5} b^{-2.5} + \frac{1}{2.5} N^{-2.5}$$

$$= \frac{1}{2.5} N^{-2.5}$$

$$\frac{1}{2.5} N^{-2.5} \le 0.02$$

$$\frac{1}{2.5} \le 0.02 N^{2.5}$$

$$\frac{1}{2.5(0.02)} \le N^{2.5}$$

$$\left[\frac{1}{2.5(0.02)}\right]^{\frac{1}{2.5}} \le N$$

$N \geq 3.3$

$\boxed{N = 4}$

Ex: Revolve the curve about the x-axis. Surface area?

$$\begin{cases} x = 1 + t \\ y = 2\sqrt{t} \\ 0 \le t \le 4 \end{cases}$$

$$S_x = 2\pi \int_a^b y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = t^{-1/2}$$

$$S_x = 2\pi \int_0^4 2\sqrt{t} \sqrt{1 + t^{-1}} \, dt$$

$$= 4\pi \int_0^4 \left[t + 1 \right]^{1/2} dt$$

$$\boxed{\begin{array}{l} u = t+1 \\ du = dt \\ t=0 \Rightarrow u=1 \\ t=4 \Rightarrow u=5 \end{array}}$$

$$= 4\pi \int_1^5 u^{1/2} \, du$$

$$= 4\pi \left[\frac{2}{3} u^{3/2} \right]_1^5$$

$$= \frac{8\pi}{3} \left[5^{3/2} - 1 \right]$$

Ex: Revolve the curve about the y-axis. Surface area?

$$\cancel{\begin{cases} x = a\sin^3\theta \\ y = a\cos^3\theta \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}}$$

$$\begin{cases} x = a\sin^3 t \\ y = a\cos^3 t \\ 0 \le t \le \frac{\pi}{2} \end{cases}$$

$$\frac{dx}{dt} = 3a\sin^2 t\cos t \qquad \frac{dy}{dt} = -3a\cos^2 t\sin t$$

$$S_y = 2\pi \int_a^b x(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\pi \int_0^{\frac{\pi}{2}} a\sin^3 t \sqrt{9a^2\sin^4 t\cos^2 t + 9a^2\cos^4 t\sin^2 t}\, dt$$

$$= 2\pi \int_0^{\frac{\pi}{2}} a\sin^3 t \sqrt{9a^2\sin^2 t\cos^2 t\,\cancel{(\sin^2 t + \cos^2 t)}}\, dt$$

$$= 2\pi \int_0^{\frac{\pi}{2}} a\sin^3 t\, |3a\sin t\cos t|\, dt$$

$$= 2\pi \int_0^{\pi/2} a\sin^3 t\,(3a\sin t\cos t)\, dt$$

$$= 6\pi a^2 \int_0^{\pi/2} \sin^4 t\cos t\, dt$$

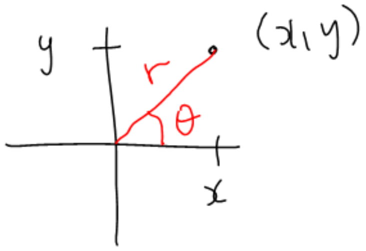
$$\boxed{\begin{array}{l} u = \sin t \\ du = \cos t\, dt \\ t = 0 \Rightarrow u = 0 \\ t = \frac{\pi}{2} \Rightarrow u = 1 \end{array}}$$

$$= 6\pi a^2 \int_0^1 u^4\, du$$

$$= 6\pi a^2 \left(\frac{u^5}{5}\right)\Bigg|_{u=0}^{u=1}$$

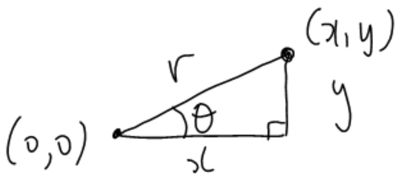
$$= \frac{6a^2\pi}{5}$$

10.4 Polar Coordinates and Graphs



Rectangular Coordinates (x,y)

Polar Coordinates (r,θ)



$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\frac{y}{x}\ (+\pi?)$

Add π when $x<0$

Ex: Convert $(r,\theta) = (6, \frac{\pi}{6})$ to rectangular.

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$x = 6\cos\frac{\pi}{6}$
$= 6\left(\frac{\sqrt{3}}{2}\right)$
$= 3\sqrt{3}$

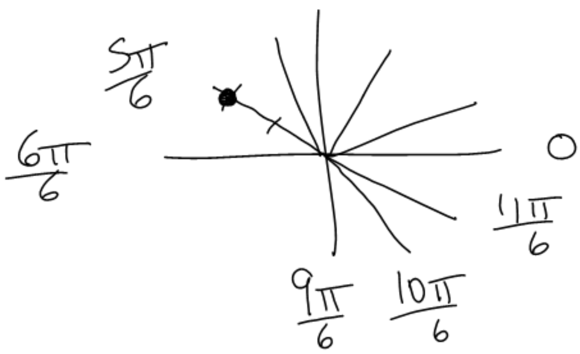
$y = 6\sin\frac{\pi}{6}$
$= 6\left(\frac{1}{2}\right)$
$= 3$

<u>Ex</u>: Convert $(x,y) = (-\sqrt{3}, 1)$ to polar.

$r = \sqrt{(-\sqrt{3})^2 + 1^2}$
$= \sqrt{4}$
$= 2$

$\theta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)$ $(+\pi ?)$
$= \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \pi$
$= -\frac{\pi}{6} + \pi$
$= \frac{5\pi}{6}$

<u>Ex</u>: Describe $(r,\theta) = \left(2, \frac{5\pi}{6}\right)$ with a negative r-value.

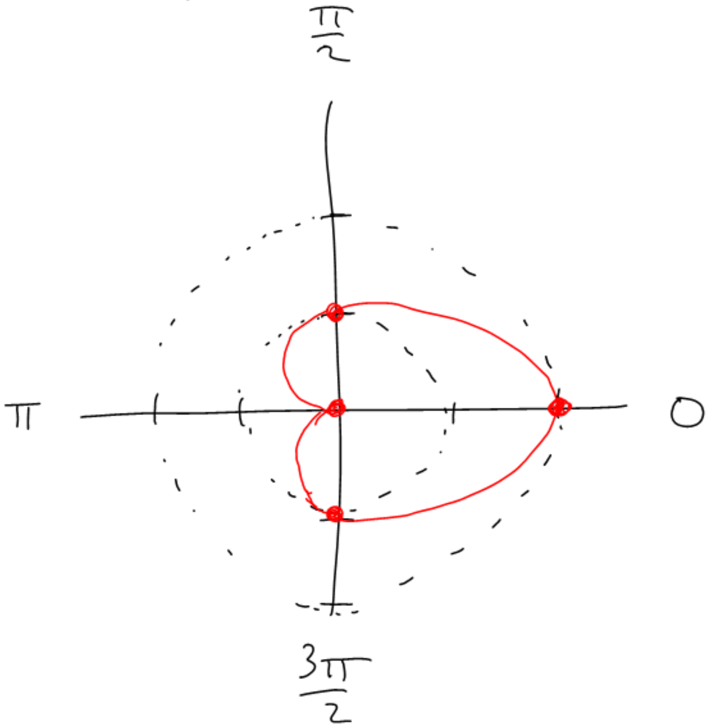


$\left(-2, \frac{11\pi}{6}\right)$

or $\left(-2, -\frac{\pi}{6}\right)$

Ex: Sketch $r = 1 + \cos\theta$

θ	$r = 1+\cos\theta$
0	2
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	1
2π	2



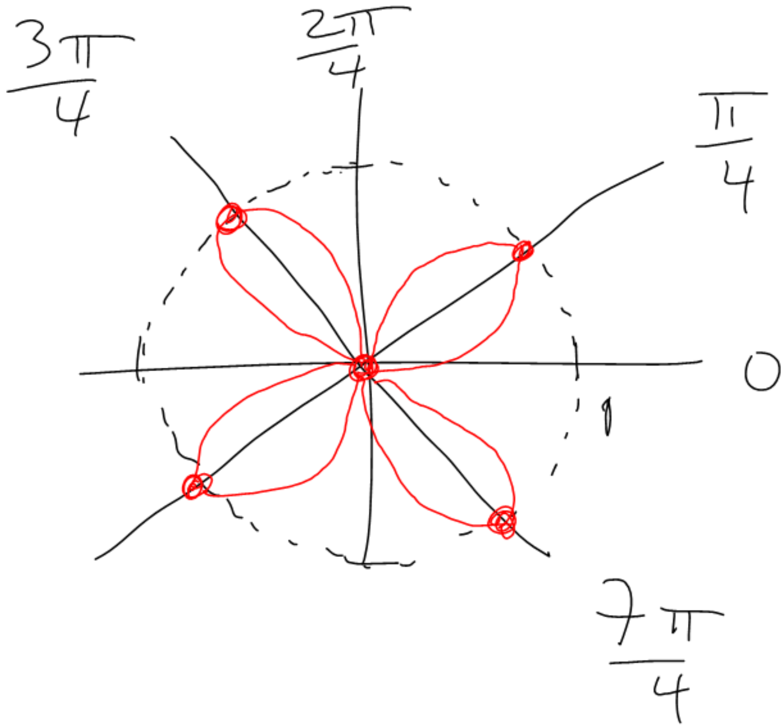
limaçon

(Pronounced LI - MA - So)

Ex: Sketch $r = \sin 2\theta$

θ	$r = \sin 2\theta$
0	0
$\frac{\pi}{4}$	1
$\frac{2\pi}{4}$	0
$\frac{3\pi}{4}$	-1
$\frac{4\pi}{4}$	0

θ	$r = \sin 2\theta$
$\frac{5\pi}{4}$	1
$\frac{6\pi}{4}$	0
$\frac{7\pi}{4}$	-1
$\frac{8\pi}{4}$	0



rose
with 4 petals