Test 3

FRI NOV 10
9.2-9.8
6 Questions
Bring: calculator, music/earplugs
Practice Problems online

Ch 10
Parametric Curves
Polar Curves

10.2 Parametric Curves

$$\begin{cases} x = f(t) \\ y = g(t) \\ a \le t \le b \end{cases}$$

is a parametric curve.

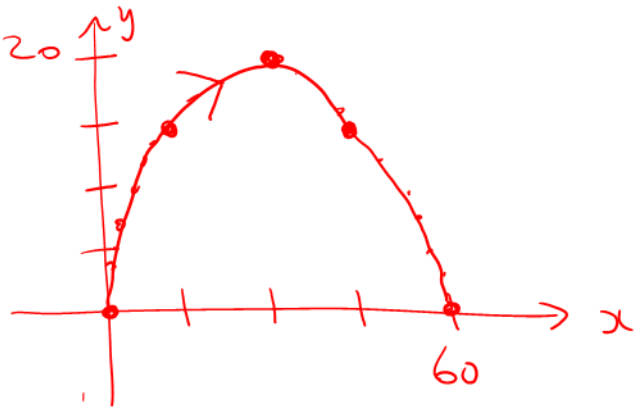
t is a parameter.
A parametric curve is a set of equations that trace the curve as t varies.

Ex: $\begin{cases} x = 15t \\ y = 20t - 5t^2 \\ 0 \le t \le 4 \end{cases}$

Plot it.

t	x	y
0	0	0
1	15	15
2	30	20
3	45	15
4	60	0

Plot



Ex: Eliminate the parameter.

$$\begin{cases} x = 15t \\ y = 20t - 5t^2 \end{cases} \quad (-\infty < t < \infty)$$

Solve for t: $x = 15t$

$$\frac{x}{15} = t$$

$t = \frac{x}{15} \rightarrow y = 20t - 5t^2$

$$y = 20\left(\frac{x}{15}\right) - 5\left(\frac{x}{15}\right)^2 \quad \checkmark$$

$$y = \frac{4x}{3} - \frac{5x^2}{225} \quad \frac{x^2}{45} \quad \checkmark$$

Ex: Eliminate the parameter

$$\begin{cases} x = h + a\cos t \\ y = k + b\sin t \end{cases} \quad 0 \le t < 2\pi$$

Solve for t: $x - h = a\cos t$ | $y - k = b\sin t$

$$\frac{x-h}{a} = \cos t \quad \Big| \quad \frac{y-k}{b} = \sin t$$

$$\boxed{\cos^2 t + \sin^2 t = 1}$$

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1 \quad \checkmark$$

ELLIPSE

Ex: Write as a parametric curve

$$(x-2)^2 + (y+5)^2 = 9$$

$$\frac{(x-2)^2}{9} + \frac{(y+5)^2}{9} = 1$$

$$\left(\frac{x-2}{3}\right)^2 + \left(\frac{y+5}{3}\right)^2 = 1$$

Let $\frac{x-2}{3} = \cos t$ | $\frac{y+5}{3} = \sin t$

$x-2 = 3\cos t$ | $y+5 = 3\sin t$

$x = 2+3\cos t$ | $y = -5+3\sin t$

$$\begin{cases} x = 2+3\cos t \\ y = -5+3\sin t \\ 0 \le t < 2\pi \end{cases}$$

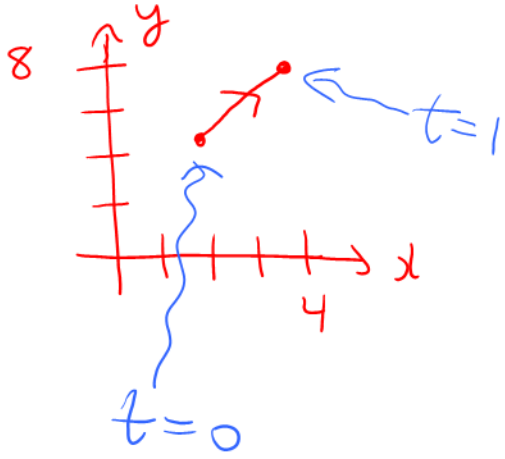
Parametrization of a curve is not unique.

Ex: Give 3 parametrizations of the line segment from (2,5) to (4,8).

a) direction vector

$$\vec{d} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + t\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



$$\begin{cases} x = 2 + 2t \\ y = 5 + 3t \\ 0 \le t \le 1 \end{cases}$$

b) Speed it up.

$$\begin{cases} 0 \le t \le 0.5 \\ x = 2 + \underline{\underline{4t}} \\ y = 5 + \underline{\underline{6t}} \end{cases}$$

c) Slow it down.

$$\begin{cases} 0 \le t \le 2 \\ x = 2 + t \\ y = 5 + \frac{3}{2}t \end{cases}$$