Section 1.3 #51 $\lim_{x \to -3} \frac{x^2+x-6}{x^2-9}$

$= \lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)(x-3)}$

$= \frac{-5}{-6}$

$= \frac{5}{6}$

Section 1.5 #51 $\lim_{x \to 1^+} \frac{x^2+x+1}{x^3-1}$

$= \frac{3}{0^+}$

$= \infty$

Section 2.3 #49

Find y' for $y = \frac{3(1-\sin x)}{2\cos x}$

$y' = \frac{2\cos x[3(-\cos x)] - 3(1-\sin x)(-2\sin x)}{4\cos^2 x}$

$= \frac{-6\cos^2 x + 6\sin x(1-\sin x)}{4\cos^2 x}$

$$= \frac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{4\cos^2 x}$$

$$= \frac{-6 + 6\sin x}{4\cos^2 x}$$

Section 2.4 #19

Find $g'(x)$ if $g(x) = \frac{6}{(x^3-2)^3}$

$$g(x) = 6(x^3-2)^{-3}$$

$$g'(x) = -18(x^3-2)^{-4}(3x^2)$$

$$= \frac{-54x^2}{(x^3-2)^4}$$

5.1 Derivatives of Exponentials and Logs Cont'd

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

Ex: Find y'

d) $y = \ln \sqrt{\frac{x+1}{2x+3}}$

$$y = \frac{1}{2} \ln \left(\frac{x+1}{2x+3} \right)$$

$$= \frac{1}{2} \left[\ln(x+1) - \ln(2x+3) \right]$$

$$y' = \frac{1}{2} \left[\frac{1}{x+1} - \frac{2}{2x+3} \right]$$

e) $y = e^{x^2} + \cos(\ln x)$

$$y' = e^{x^2}(2x) - \sin(\ln x)\frac{1}{x}$$

$$= 2xe^{x^2} - \frac{1}{x}\sin(\ln x)$$

Ex: Let $y = e^{2x} - 5x$

Find x so that $y' = 0$.

$$y' = e^{2x}(2) - 5$$

$$= 2e^{2x} - 5$$

Set $y' = 0$: $\quad 2e^{2x} - 5 = 0$

$$2e^{2x} = 5$$

$$e^{2x} = \frac{5}{2}$$

Take ln: $\quad 2x = \ln\left(\frac{5}{2}\right)$

$$x = \frac{1}{2}\ln\left(\frac{5}{2}\right)$$

2.5 Implicit Differentiation

y is an <u>explicit</u> function of x:

$$y = \pm\sqrt{25 - x^2}$$

y is an <u>implicit</u> function of x:

$$x^2 + y^2 = 25$$

<u>Ex:</u> y depends on x

Find:

a) $\frac{d}{dx}[x^2] = 2x$

b) $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$ Chain Rule

c) $\frac{d}{dx}[(4x^2)y^3]$

$= (4x^2)(3y^2\frac{dy}{dx}) + y^3(8x)$

ASIDE

$$\frac{d}{dx}\left(x^3+1\right)^2 = 2\left(x^3+1\right)\left(3x^2\right)$$

$$\frac{d}{dx}\, y^2 = 2y\,\frac{dy}{dx}$$

$$\frac{d}{dx}\, x^2 = 2x$$

Ex: Find $\frac{dy}{dx}$ given $x^2+y^2=25$

1) Take $\frac{d}{dx}$

$$2x + 2y\frac{dy}{dx} = 0$$

2) Solve for $\frac{dy}{dx}$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} \text{ or } \frac{-x}{y}$$