Suggested HW pdf is on D2L
(Answers @ back)

List of problems on website

Test 2 ~~Fri Oct 20~~ Mon Oct 23

Test 4 ~~Fri Dec 1~~ Mon Dec 4

2.2 - 2.4 Derivative Rules and Trig Cont'd

Power Rule $\frac{d}{dx}[x^n] = nx^{n-1}$

Product Rule $[fg]' = fg' + gf'$

Quotient Rule $\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$

Chain Rule: Calculation Version

$$[f(g(x))]' = f'(g(x))\, g'(x)$$

Chain Rule : Formal Version

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

Ex: $y = \sqrt[3]{x^3+1}$. Find y'.

$y = (x^3+1)^{1/3}$

$y' = \frac{1}{3}(x^3+1)^{-2/3}(3x^2)$ ✓

$= \dfrac{x^2}{\sqrt[3]{(x^3+1)^2}}$ ✓

Ex: Confirm using the formal Chain Rule.

$y = \sqrt[3]{u}$ $\quad u = x^3+1$

$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$= \frac{1}{3}u^{-2/3}(3x^2)$

$= \frac{1}{3}(x^3+1)^{-2/3}(3x^2)$ ✓

Ex: Find $\dfrac{dy}{dx}$

a) $y = 5x^4 + \dfrac{3}{x^2} + 6\sqrt{x}$

$y = 5x^4 + 3x^{-2} + 6x^{1/2}$

$$\frac{dy}{dx} = 20x^3 - 6x^{-3} + 3x^{-1/2}$$

b) $y = \dfrac{x^3}{2x+1}$

$$\frac{dy}{dx} = \frac{(2x+1)(3x^2) - x^3(2)}{(2x+1)^2}$$

$$= \frac{4x^3 + 3x^2}{(2x+1)^2}$$

Ex: Find $f'(1)$ for

$f(x) = x^2(x^2+5x+1)(x^7+x^3+6)$

$f(x) = (x^4+5x^3+x^2)(x^7+x^3+6)$

$$f'(x) = (x^4+5x^3+x^2)(7x^6+3x^2) + (x^7+x^3+6)(4x^3+15x^2+2x)$$

$$f'(1) = (7)(10) + (8)(21)$$

$$= 238$$

Aside $f(x) = \frac{x^3}{2x+1}$

$f(x) = x^3(2x+1)^{-1}$

Product Rule ✓

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$

Ex: Find y'

a) $y = \csc x^2$

$y' = -\csc x^2 \cot x^2 \; (2x)$

$= -2x \csc x^2 \cot x^2$

b) $y = \csc^2 x$

$y = [\csc x]^2$

$y' = 2[\csc x](-\csc x \cot x)$

$= -2\csc^2 x \cot x$

<u>Ex</u>: Why does $\frac{d}{dx}[\tan x] = \sec^2 x$?

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \left[\frac{1}{\cos x}\right]^2$$

$$= \sec^2 x$$

Ex: Find $\frac{df}{dx}$

a) $f = \sin 5x$

$$\frac{df}{dx} = \cos 5x \ (5)$$

$$= 5\cos 5x$$

b) $f = x \tan x^2$

$$\frac{df}{dx} = x\left[\sec^2 x^2 \ (2x)\right] + \left(\tan x^2\right)(1)$$

$$= 2x^2 \sec^2 x^2 + \tan x^2$$

c) $f = \sin^3 x + \cos^3 x$

$$f = [\sin x]^3 + [\cos x]^3$$

$$\frac{df}{dx} = 3[\sin x]^2 (\cos x) + 3[\cos x]^2(-\sin x)$$

$$= 3\sin^2 x \cos x - 3\cos^2 x \sin x$$

d) $f = \sec^2 x^3$

$$f = [\sec x^3]^2$$

$$\frac{df}{dx} = 2[\sec x^3][\sec x^3 \tan x^3 (3x^2)]$$

$$= 6x^2 \sec^2 x^3 \tan x^3$$

<u>Ex</u>: Find the second derivative of $y = \sin 3x^2$

$$y' = \cos 3x^2 (6x)$$

$$= \underbrace{6x}\,\underbrace{\cos 3x^2}$$

$$y'' = 6x[-\sin 3x^2 (6x)] + (\cos 3x^2)(6)$$

$$= -36x^2 \sin 3x^2 + 6\cos 3x^2$$

<u>Ex</u>: Find an equation for the tangent line to $y = 2x^3 + 5x^2 - 1$ at $x = 1$.

$$y' = 6x^2 + 10x$$

$$y'\Big|_{x=1} = 16$$

$m = 16$, $x_1 = 1$, $y_1 = 6$

$$y - y_1 = m(x - x_1)$$

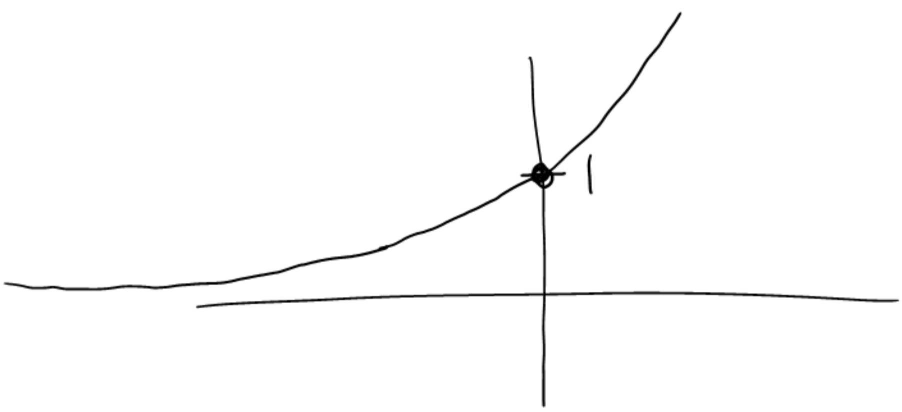
$$y - 6 = 16(x - 1)$$

5.1 Derivatives of Exponentials and Logs

Exponential Function:

$f(x) = b^x$ b: constant

The most important base is $e \approx 2.718$



$y = e^x$

$$\frac{d}{dx} e^{x} = e^{x}$$

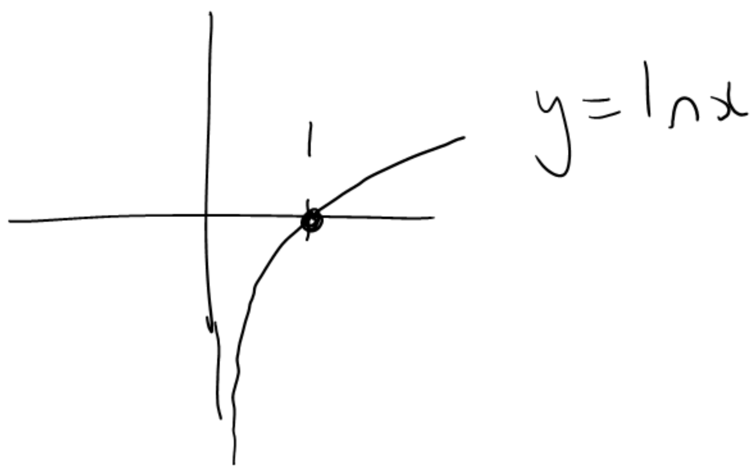
$$\frac{d}{dx} e^{u} = e^{u} \frac{du}{dx}$$

$$\boxed{\begin{aligned} e^{a} e^{b} &= e^{a+b} \\ \frac{e^{a}}{e^{b}} &= e^{a-b} \\ (e^{a})^{b} &= e^{ab} \end{aligned}}$$

Logarithmic Function:

$f(x) = \log_b x$ $\quad$ b: constant

Note: $\log_e x$ is written $\ln x$



Ex: Find $\lim_{x \to 7^+} \ln(x-7)$

$= \ln 0^+$

$= -\infty$

$$\boxed{\begin{aligned} \frac{d}{dx} \ln x &= \frac{1}{x} \\ \frac{d}{dx} \ln u &= \frac{1}{u} \frac{du}{dx} \end{aligned}}$$

$$\boxed{\begin{aligned} \ln(ab) &= \ln a + \ln b \\ \ln\left(\frac{a}{b}\right) &= \ln a - \ln b \\ \ln a^b &= b \ln a \end{aligned}}$$

Ex: Find y'

a) $y = e^{3x}$

$y' = e^{3x}(3)$

$= 3e^{3x}$

b) $y = \ln(x^3+1)$

$y' = \frac{1}{x^3+1}(3x^2)$

$= \frac{3x^2}{x^3+1}$

c) $y = e^{-2x} \sin 5x$

$y' = e^{-2x}[5\cos 5x]$

$+ (\sin 5x)[-2e^{-2x}]$

$= 5e^{-2x}\cos 5x - 2e^{-2x}\sin 5x$

or $e^{-2x}(5\cos 5x - 2\sin 5x)$