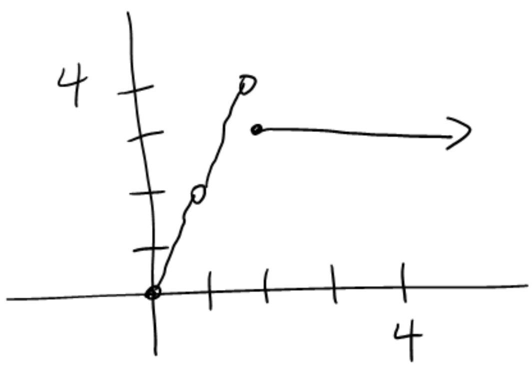
Math 250A Website:
www.leahhoward.com

Suggested HW is on D2L
List of which problems to do is on website
Answers at back of pdf file.

Thursday class 9:30-10:50 in TEC 174

1.2-1.5 Limits and Continuity



$y = f(x)$

$$\lim_{x \to 1} f(x) = 2$$

Means: As x approaches 1, $f(x)$ approaches 2.
Behaviour at $x=1$ is irrelevant.

$$\lim_{x \to 4} f(x) = 3$$

$\lim_{x \to 2} f(x)$ does not exist

Limit from the left $\lim_{x \to 2^-} f(x) = 4$

" right $\lim_{x \to 2^+} f(x) = 3$

"One-sided limits"

Def

$f(x)$ is continuous at $x=a$ if

$$\lim_{x \to a} f(x) = f(a)$$

[no jump/hole in graph at $x=a$]

Def

$f(x)$ is continuous on an interval if $f(x)$ is continuous at all x-values in the interval.

Ex: Find $\lim_{x \to 2} \frac{2x+1}{x+1}$

$= \frac{5}{3}$

Can plug in when the function is continuous at the x-value.

$\underline{\text{Ex}}$: Find $\lim_{x\to 2} \frac{x^2-4}{x^2-x-2}$ $\left(\frac{0}{0} \text{ no info}\right)$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)}$$

$$= \frac{4}{3}$$

Recall $a^3-b^3 = (a-b)(a^2+ab+b^2)$

$\underline{\text{Ex}}$: Find $\lim_{x\to 2} \frac{x^3-8}{x^4-16}$ $\left(\frac{0}{0} \text{ no info}\right)$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x^2-4)(x^2+4)}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)(x^2+4)}$$

$$= \frac{12}{32}$$

$\underline{\text{Ex}}$: Find $\lim_{x\to 6} \frac{\sqrt{x+3}-3}{x-6}$ $\left(\frac{0}{0} \text{ no info}\right)$

$$= \lim_{x\to 6} \frac{(\sqrt{x+3}-3)}{(x-6)} \cdot \frac{(\sqrt{x+3}+3)}{(\sqrt{x+3}+3)}$$

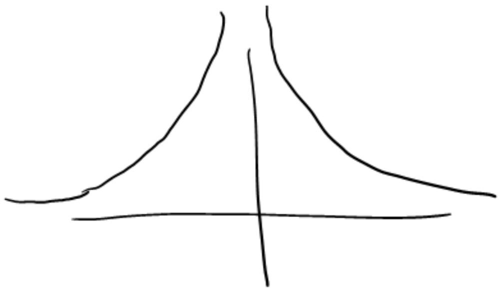
$$= \lim_{x\to 6} \frac{x+3-9}{(x-6)(\sqrt{x+3}+3)}$$

$$= \lim_{x \to 6} \frac{\overset{1}{\cancel{x-6}}}{\cancel{(x-6)}(\sqrt{x+3} + 3)}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$

<u>Ex</u>: Find $\lim_{x \to 0} \frac{1}{x^2}$ and $\lim_{x \to -\infty} \frac{1}{x^2}$



$y = \frac{1}{x^2}$

$\lim_{x \to 0} \frac{1}{x^2}$ is undefined

$\lim_{x \to 0} \frac{1}{x^2} = \infty$ (more precise)

$\lim_{x \to -\infty} \frac{1}{x^2} = 0$

<u>Ex</u>: Find $\lim_{x \to 3^+} \frac{2x}{9-x^2}$

$$= \frac{6}{0^-}$$

$$= -\infty$$

Fact

$$\lim_{x\to\infty} \frac{1}{x^N} = 0 \quad \text{and} \quad \lim_{x\to-\infty} \frac{1}{x^N} = 0$$

if $N > 0$.

Ex: Find $\displaystyle\lim_{x\to\infty} \frac{3x^2+5x+1}{5x^2+7}$

$\left(\frac{\infty}{\infty} \text{ no info}\right)$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}\left(3+\frac{5}{x}+\frac{1}{x^2}\right)}{\cancel{x^2}\left(5+\frac{7}{x^2}\right)}$$

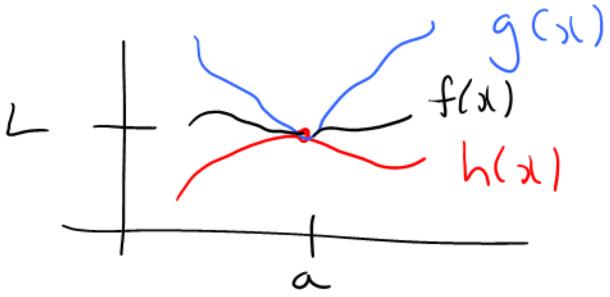
$$= \frac{3}{5}$$

Squeeze Theorem

Suppose $h(x) \le f(x) \le g(x)$
for all x near $x=a$.

If $\displaystyle\lim_{x\to a} h(x) = L$ and $\displaystyle\lim_{x\to a} g(x) = L$

then $\displaystyle\lim_{x\to a} f(x) = L$.



Ex: Find $\lim_{x\to 0} x^2 \sin\frac{1}{x}$

KEY OBSERVATION: $-1 \le \sin\frac{1}{x} \le 1$

$$-x^2 \le x^2\sin\frac{1}{x} \le x^2$$

$$\lim_{x\to 0} -x^2 = 0 \quad \text{and} \quad \lim_{x\to 0} x^2 = 0$$

Conclude $\lim_{x\to 0} x^2\sin\frac{1}{x} = 0$

2.2-2.4 Derivative Rules and Trig

The derivative of $y = f(x)$ can be written:

$$y', \quad \frac{dy}{dx}, \quad f'(x), \quad \frac{df}{dx}$$

Evaluating:

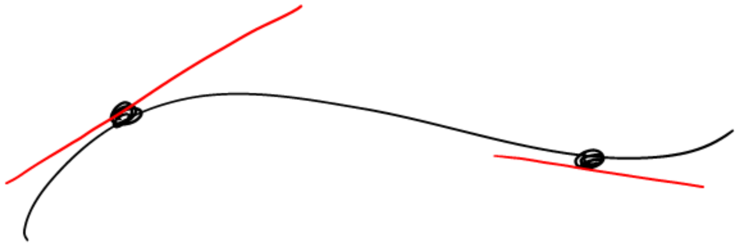
$$y'\Big|_{x=a}, \quad \frac{dy}{dx}\Big|_{x=a}, \quad f'(a), \quad \frac{df}{dx}\Big|_{x=a}$$

$f'(x)$ represents:

slope of the tangent line to $y = f(x)$

AND

(instantaneous) rate of change of $f(x)$



$y = f(x)$