Test 3
FRI Nov 10
9.2–9.8 (6 Questions)
Bring calculator
Bring music/earplugs
Practice Problems on website

9.7 Taylor Polynomials Cont'd

Ex: Find N so that the Maclaurin polynomial $P_N(x)$ approximates e^{-1} with error less than 0.001

$f(x) = e^x$

$f^{(n)}(x) = e^x$

$f^{(N+1)}(x) = e^x$

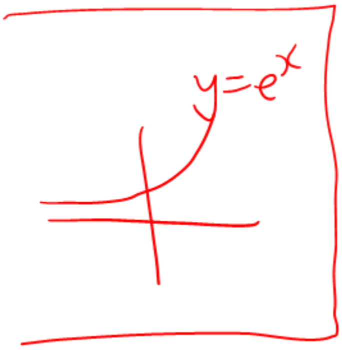
$f^{(N+1)}(z) = e^z$

$c = 0$

$x = -1$

$$|R_N(x)| = \left| \frac{f^{(N+1)}(z)}{(N+1)!} (x-c)^{N+1} \right|$$

$$= \left| \frac{e^z}{(N+1)!} (-1)^{N+1} \right|$$



$$= \frac{e^z}{(N+1)!}$$

where z is between -1 and 0

$$\leq \frac{e^0}{(N+1)!}$$

$$\leq \frac{1}{(N+1)!}$$

Want $\frac{1}{(N+1)!} < 0.001$

Guess and check

$N=1$ ✗

$N=2$ ✗

$N=3$ ✗

$N=4$ ✗

$N=5$ ✗

$N=6$ ✓

$$\boxed{N \geq 6}$$

9.8 Power Series

Power series centred at c :

$$\sum_{n=0}^{\infty} b_n (x-c)^n = b_0 + b_1(x-c) + b_2(x-c)^2 + \dots$$

Note: b_0, b_1, b_2 are the coefficients

See handout on website.

Ex: Find the interval of convergence:

$$\sum_{n=1}^{\infty} \underbrace{\frac{(-1)^n (x-3)^n}{n \cdot 5^n}}_{a_n}$$

Use ratio test.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x-3)^{n+1}}{(n+1)\cdot 5^{n+1}} \cdot \frac{n \cdot 5^n}{(x-3)^n}\right|$$

$$= \left|\frac{x-3}{5} \cdot \frac{n}{n+1}\right|$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x-3}{5}(1)\right|$$

l'Hôpital's Rule

$$= \left|\frac{x-3}{5}\right|$$

Series converges if $\left|\frac{x-3}{5}\right| < 1$

$$|x-3| < 5$$

centre of series

radius of convergence

$$-5 < x-3 < 5$$

$$-2 < x < 8$$

$x=-2$: $$\sum_{n=1}^{\infty} \frac{\cancel{(-1)^n (-5)^n}^{\,5^n}}{n \cdot 5^n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

Diverges (p-series)

$x=8$: $$\sum_{n=1}^{\infty} \frac{(-1)^n (\cancel{5})^n}{n \cdot \cancel{5}^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Converges (Alternating Series Test)

$$\boxed{\text{Interval of Convergence: } -2 < x \leq 8}$$