Test MON OCT 23

8.2-8.5, 5.6, 8.8, 9.1

<u>Ex</u>: Evaluate $\lim_{x \to 0^+} (e^x + x)^{1/x}$

$$L = \lim_{x \to 0^+} (e^x + x)^{1/x}$$

$$\ln L = \lim_{x \to 0^+} \ln (e^x + x)^{1/x}$$

$$= \lim_{x \to 0^+} \frac{\ln(e^x + x)}{x}$$

$\frac{0}{0}$

$$\overset{H}{=} \lim_{x \to 0^+} \frac{\frac{1}{e^x + x}(e^x + 1)}{1}$$

$$= 2$$

$$\ln L = 2 \Rightarrow L = e^{\ln L} = e^2$$

Ex: Evaluate, if possible:

$$\int_0^1 \frac{dx}{2x-1} \Leftarrow \text{Asymptote at } x=\tfrac{1}{2}$$

$$= \underbrace{\int_0^{1/2} \frac{dx}{2x-1}}_{I_1} + \underbrace{\int_{1/2}^{1} \frac{dx}{2x-1}}_{I_2}$$

$$I_1 = \int_0^{1/2} \frac{dx}{2x-1}$$

$$= \lim_{t\to\frac{1}{2}^-} \int_0^t \frac{dx}{2x-1}$$

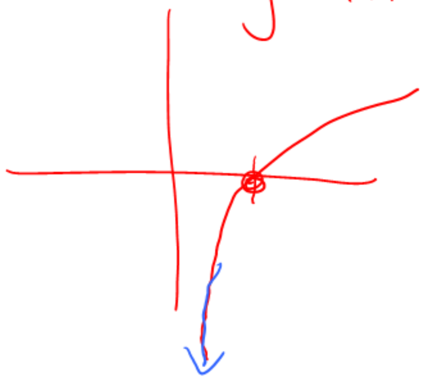
$$= \lim_{t\to\frac{1}{2}^-} \frac{1}{2}\ln|2x-1| \Big|_0^t$$

Shortcut $\displaystyle\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$

$$= \lim_{t \to \frac{1}{2}^-} \frac{1}{2} \ln |2t-1| - \frac{1}{2} \ln 1$$

$$= \frac{1}{2} \ln 0^+$$

$$= -\infty$$



I_1 diverges

$\Rightarrow$ Integral diverges

Ex: Find $\int \sqrt{4-x^2}\, dx$

Trig Sub $\quad x = 2\sin\theta$

$\qquad dx = 2\cos\theta\, d\theta$

$$\frac{x}{2} = \sin\theta$$

$$\begin{cases} a^2+x^2=4 \\ \vdots \\ a=\sqrt{4-x^2} \end{cases}$$

$$\frac{\sqrt{4-x^2}}{2} = \cos\theta$$

$$\sqrt{4-x^2} = 2\cos\theta$$

$$\int \sqrt{4-x^2}\,dx = \int 2\cos\theta\,(2\cos\theta\,d\theta)$$

$$= \int 4\cos^2\theta\,d\theta$$

$$= \int 4\,\frac{(1+\cos 2\theta)}{2}\,d\theta$$

$$= 2\left[\theta + \frac{\overset{2\sin\theta\cos\theta}{\cancel{\sin 2\theta}}}{2}\right] + C$$

$$= 2\left[\theta + \sin\theta\cos\theta\right] + C$$

$$= 2\left[\sin^{-1}\frac{x}{2} + \frac{x}{2}\,\frac{\sqrt{4-x^2}}{2}\right] + C$$

$$= 2\sin^{-1}\frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} + C$$

Ex: Find $\displaystyle\int \frac{4}{(x+3)(x^2+9)}\,dx$

$$\frac{4}{(x+3)(x^2+9)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+9}$$

$$4 = A(x^2+9) + (Bx+C)(x+3)$$

Sub $x=-3$: $\quad 4 = A(18) \Rightarrow A = \frac{2}{9}$

x^2 coefficient: $\quad 0 = A + B \Rightarrow B = \frac{-2}{9}$

Sub $x=0$: $\quad 4 = 9A + 3C$

$$4 = 2 + 3C$$

$$2 = 3C$$

$$C = \frac{2}{3}$$

$$\text{Integral} = \int \left[\frac{2}{9}\frac{1}{(x+3)} - \frac{2}{9}\frac{x}{(x^2+9)} + \frac{2}{3}\frac{1}{(x^2+9)}\right]dx$$

$$= \frac{2}{9}\ln|x+3| - \frac{1}{9}\ln|x^2+9|$$
$$+ \quad \frac{2}{9}\tan^{-1}\frac{x}{3} \quad + C$$

Ex: Find $\int \sin^3 2x \cos^8 2x \, dx$

WANT:

$\int$ <powers of $\sin\theta$> $\cos\theta \, d\theta$

$\int$ <powers of $\cos\theta$> $\sin\theta \, d\theta$

$\int$ <powers of $\tan\theta$> $\sec^2\theta \, d\theta$

etc.

$$= \int (\sin 2x)\underbrace{\cancel{(\sin^2 2x)}}_{(1-\cos^2 2x)} \cos^8 2x \, dx$$

$$\boxed{\begin{aligned} u &= \cos 2x \\ du &= -2\sin 2x \, dx \\ -\tfrac{1}{2} du &= \sin 2x \, dx \end{aligned}}$$

$$= -\frac{1}{2}\int (1-u^2)\,u^8 \quad du$$

$$= -\frac{1}{2}\int (u^8 - u^{10})\,du$$

$$= -\frac{1}{2}\left[\frac{u^9}{9} - \frac{u^{11}}{11}\right] + C$$

$$= -\frac{1}{2}\left[\frac{\cos^9 2x}{9} - \frac{\cos^{11} 2x}{11}\right] + C$$