

9.5 Alternating Series

In an alternating series, signs alternate.

Examples:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

Alternating Series Test

Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

both converge if $\lim_{n \rightarrow \infty} a_n = 0$

and $a_{n+1} \leq a_n$ for all n .

Ex: Test for convergence: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$

It's an alternating series \checkmark

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{2n-1} \leftarrow a_n$$
$$= 0 \quad \checkmark$$

$$a_{n+1} = \frac{1}{2(n+1)-1}$$
$$= \frac{1}{2n+1}$$
$$\leq \frac{1}{2n-1} \quad \text{for all } n. \quad \checkmark$$

The series converges.

Recall:

Partial Sum $S_N =$ sum of first N terms

$S =$ sum of the series

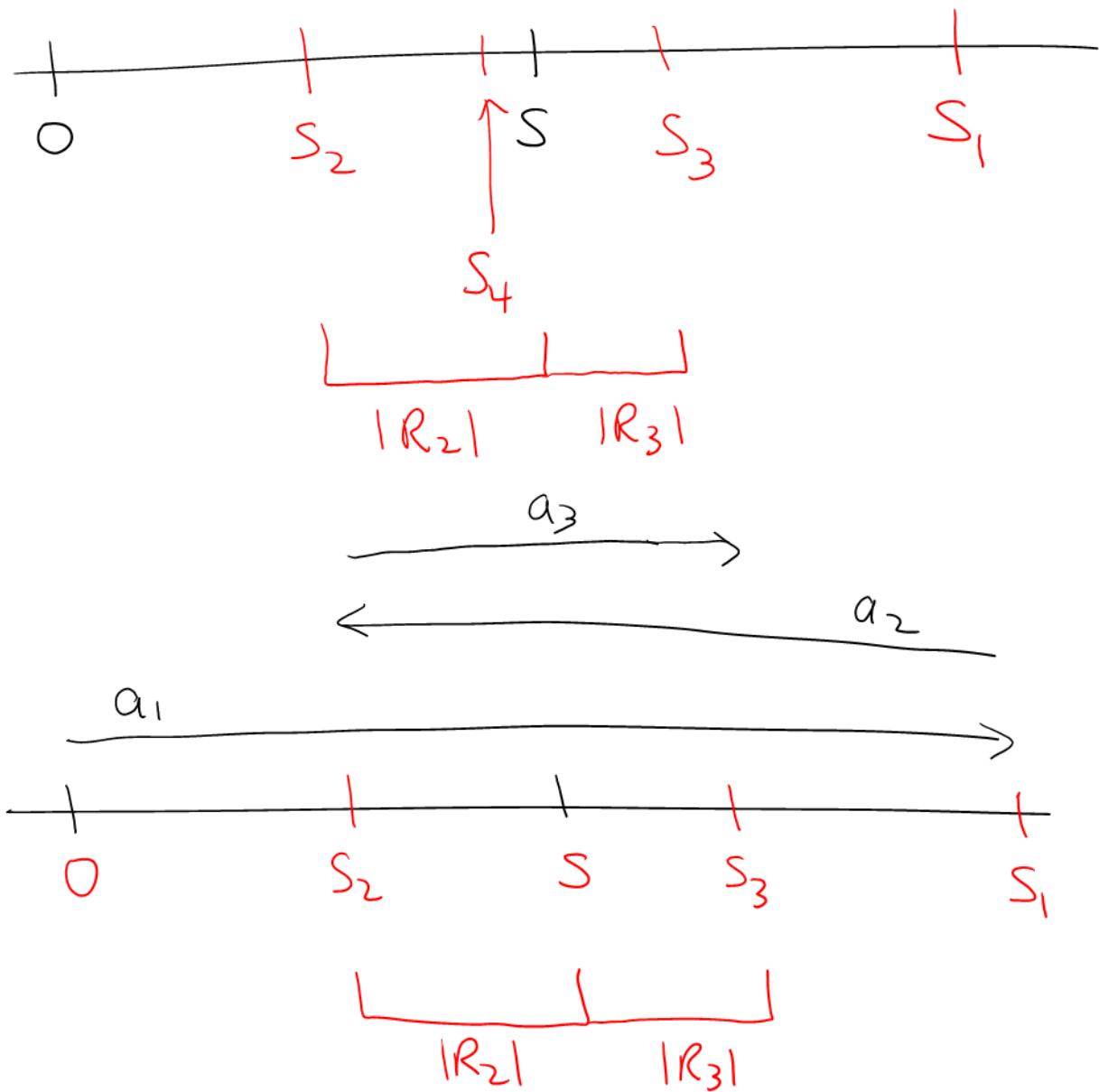
Remainder/Error $R_N = S - S_N$

FACT

If a series converges by the Alternating Series Test then

$$|R_N| \leq a_{N+1}$$

Why?



$$|R_2| \leq a_3$$

$$|R_3| \leq a_4$$

$$|R_4| \leq a_5$$

\vdots

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges by the
Alternating Series Test.

a) Calculate S_{19}

$$S_{19} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{19} \\ \approx 0.7188$$

b) Find an upper bound for $|R_{19}|$

$$|R_{19}| \leq a_{20} \\ \leq \frac{1}{20} \\ \leq 0.05$$

c) Estimate S

$$0.7188 - 0.05 \leq S \leq 0.7188 + 0.05$$

Extra: a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n+1}$

Does it converge?

Alternating Series ✓

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{4n+1} = 0 \quad \checkmark$$

$$a_{n+1} = \frac{1}{4(n+1)+1} \leq \frac{1}{4n+1} \quad \text{for all } n \quad \checkmark$$

Yes

$$b) \sum_{n=1}^{\infty} \frac{(-1)^n n}{4n+1}$$

Does it converge?

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{4n+1} = \frac{1}{4}$$

Series diverges by n^{th} term test