Test 2

Mon Oct 23

8.2-8.5, 5.6, 8.8, 9.1 (7 Questions)

Bring calculator, music/earplugs

Practice Problems on website

9.2 Series and Convergence

Sequence: $a_0, a_1, a_2, \ldots$ (LIST)

Series: $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots$ (SUM)

The N^{th} partial sum of a series is the sum of the first N terms. Written S_N.

Ex: Find the partial sums S_1, S_2, S_3 for $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

Notation: Let $S = \lim_{N\to\infty} S_N$ (if it exists)

Def

If $S = \lim_{N\to\infty} S_N$ exists and is a real number then the series converges to S. Otherwise the series diverges.

A geometric series is:

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \dots$$

FACT

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad \text{if} \quad -1 < r < 1$$

$$\sum_{n=0}^{\infty} ar^n \text{ diverges if } |r| \geq 1$$

Ex: Let $-1 < r < 1$. Consider $\sum_{n=0}^{\infty} ar^n$.

Show $S = \frac{a}{1-r}$.

$$S_N = a + ar + ar^2 + \dots + ar^{N-1}$$

$$rS_N = \quad ar + ar^2 + \quad + ar^{N-1} + ar^N$$

$$S_N - rS_N = a - ar^N$$

$$S_N(1-r) = a - ar^N$$

$$S_N = \frac{a - ar^N}{1-r}$$

$$S = \lim_{N \to \infty} S_N$$

$$= \lim_{N \to \infty} \frac{a - ar^N}{1-r}$$

$$= \frac{a}{1-r}$$

Ex: Find the sum or show that it diverges.

a) $\sum_{n=0}^{\infty} \frac{3}{4^n}$

$= 3 + \frac{3}{4} + \frac{3}{16} + \dots$

Geometric Series $a = 3$ $r = \frac{1}{4}$ "ratio"

$= \frac{a}{1-r}$

$= \frac{3}{\left(\frac{3}{4}\right)}$

$= 3 \times \frac{4}{3}$

$= 4$

b) $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

$= 1 + \frac{3}{2} + \frac{9}{4} + \dots$

Geometric Series $a = 1$ $r = \frac{3}{2}$

The series diverges.

c) $\sum_{n=2}^{\infty} \frac{7(3^{n-1})}{5^n}$

$= \frac{21}{25} + \frac{63}{125} + \dots$

Geometric Series $a = \frac{21}{25}$ $r = \frac{3}{5}$

$= \frac{a}{1-r}$

$$= \frac{\left(\frac{21}{25}\right)}{\left(\frac{2}{5}\right)}$$

$$= \frac{21}{5\cancel{25}} \times \frac{\cancel{5}}{2}$$

$$= \frac{21}{10}$$

d) $\sum_{n=k}^{\infty} ar^n$ given $-1 < r < 1$.

$$= ar^k + ar^{k+1} + \ldots$$

Geometric Series 1st term $= ar^k$ ratio $= r$

$$= \frac{\text{1st term}}{1 - \text{ratio}}$$

$$= \frac{ar^k}{1-r}$$

Extra: Write $0.\overline{4}$ as a fraction

$0.\overline{4} = 0.4444\ldots$

$= 0.4 + 0.04 + 0.004 + \ldots$

Geometric Series $a = 0.4$ $r = 0.1$

$$= \frac{a}{1-r}$$

$$= \frac{0.4}{0.9} \quad \text{or} \quad \frac{4}{9}$$