

Test Review

Ex: Find $\lim_{x \rightarrow 0} \frac{(\sqrt{x+5} - \sqrt{5})}{x} \frac{(\sqrt{x+5} + \sqrt{5})}{(\sqrt{x+5} + \sqrt{5})}$

$$= \lim_{x \rightarrow 0} \frac{x+5 - 5}{x(\sqrt{x+5} + \sqrt{5})}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{x} 1}{\cancel{x} (\sqrt{x+5} + \sqrt{5})}$$

$$= \frac{1}{2\sqrt{5}}$$

Ex: Find $\frac{dy}{dx}$ given:

$$(\cancel{x^2} y^2) - 9x^2 - 4y^2 = 0$$

$$x^2 (2y \frac{dy}{dx}) + y^2 (2x) - 18x - 8y \frac{dy}{dx} = 0$$

$$2x^2 y \frac{dy}{dx} - 8y \frac{dy}{dx} = 18x - 2xy^2$$

$$[2x^2 y - 8y] \frac{dy}{dx} = 18x - 2xy^2$$

$$\frac{dy}{dx} = \frac{18x - 2xy^2}{2x^2 y - 8y}$$

Ex: Find $f'(x)$

$$a) f(x) = 6 \arctan x^8 - 2 \arcsin x^7 + x^3 \sec^2 x$$

$$f'(x) = 6 \cdot \frac{1}{1+(x^8)^2} \cdot 8x^7 - 2 \frac{1}{\sqrt{1-(x^7)^2}} \cdot 7x^6$$

$$+ x^3 [2 \sec x \sec x \tan x] + \sec^2 x (3x^2)$$

$$= \frac{48x^7}{1+x^{16}} - \frac{14x^6}{\sqrt{1-x^{14}}}$$

$$+ 2x^3 \sec^2 x \tan x + 3x^2 \sec^2 x$$

$$b) f(x) = \frac{e^{4x}}{\ln(2x+3)-2}$$

$$f'(x) = \frac{[\ln(2x+3)-2](4e^{4x}) - e^{4x} \left[\frac{1}{2x+3} (2) \right]}{[\ln(2x+3)-2]^2}$$

$$= \frac{4e^{4x} [\ln(2x+3)-2] - \frac{2e^{4x}}{2x+3}}{[\ln(2x+3)-2]^2}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$\frac{d}{dx} \left[\frac{1}{a} \arctan \frac{x}{a} \right] = \frac{1}{a^2+x^2}$$

$$\frac{d}{dx} [\arctan x] = \frac{1}{1+x^2}$$

Ex: Find $\int \frac{1}{\sqrt{x}(1+3\sqrt{x})} dx$

$$= \frac{2}{3} \int \frac{du}{u}$$

$$= \frac{2}{3} \ln |u| + C$$

$$= \frac{2}{3} \ln |1+3\sqrt{x}| + C$$

$$u = 1 + 3\sqrt{x}$$
$$du = \frac{3}{2} x^{-1/2} dx$$
$$\frac{2}{3} du = \frac{dx}{\sqrt{x}}$$

Ex: $\int \frac{1-\sin x}{\cos x} dx$

$$= \int \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x} \right) dx$$

$$= \int (\sec x - \tan x) dx$$
$$= \ln|\sec x + \tan x| - \ln|\sec x| + C$$

Ex: $\int \cos 4x \sqrt{1 + \sin 4x} dx$

$$u = 1 + \sin 4x$$
$$du = 4 \cos 4x dx$$
$$\frac{du}{4} = \cos 4x dx$$

$$= \frac{1}{4} \int \sqrt{u} du$$
$$= \frac{1}{4} \left(\frac{2}{3} u^{3/2} \right) + C$$
$$= \frac{1}{6} (1 + \sin 4x)^{3/2} + C$$

Ex: $\int e^{\cos 2x} \sin 2x dx$

$$u = \cos 2x$$
$$du = -2 \sin 2x dx$$
$$-\frac{1}{2} du = \sin 2x dx$$

$$= -\frac{1}{2} \int e^u du$$
$$= -\frac{1}{2} e^u + C$$

$$= -\frac{1}{2} e^{652x} + C$$

Ex: $\int \frac{x^2 - 1}{x + 3} dx$

$$\begin{array}{r} x - 3 \\ (x+3) \overline{) x^2 + 0x - 1} \\ \underline{-(x^2 + 3x)} \\ -3x - 1 \\ \underline{-(-3x - 9)} \\ 8 \end{array}$$

$$\frac{x^2 - 1}{x + 3} = x - 3 + \frac{8}{x + 3}$$

$$\begin{aligned} \text{Integral} &= \int \left[x - 3 + \frac{8}{x + 3} \right] dx \\ &= \frac{x^2}{2} - 3x + 8 \ln|x + 3| + C \end{aligned}$$

Ex: $\int \frac{e^x}{\sqrt{3 - e^{2x}}} dx$

$$= \int \frac{e^x dx}{\sqrt{\sqrt{3}^2 - (e^x)^2}}$$

$$\begin{array}{l} u = e^x \\ du = e^x dx \end{array}$$

$$= \int \frac{du}{\sqrt{\sqrt{3}^2 - u^2}}$$

$$= \sin^{-1} \frac{u}{\sqrt{3}} + C$$

$$= \sin^{-1} \frac{e^x}{\sqrt{3}} + C$$