8.3 Trig Integrals Cont'd

Three other Techniques

1) Integration By Parts
2) Convert to $\sin\theta$ and $\cos\theta$
3) Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$$

Ex: $\int \sec^3\theta \, d\theta$

$$\boxed{\begin{array}{ll} u = \sec\theta & dv = \sec^2\theta \, d\theta \\ du = \sec\theta\tan\theta \, d\theta & v = \tan\theta \end{array}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta$$

$$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$$

let $I = \int \sec^3\theta\,d\theta$

$$I = \sec\theta\tan\theta - I + \ln|\sec\theta + \tan\theta|$$

$$2I = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + C_1$$

$$I = \frac{1}{2}\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right] + C_2$$

Ex: $\int \sec\theta\cot^2\theta\,d\theta$

$$= \int \frac{1}{\cos\theta}\left(\frac{\cos\theta}{\sin\theta}\right)^2 d\theta$$

$$= \int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$$\boxed{\begin{array}{c} u = \sin\theta \\ du = \cos\theta\, d\theta \end{array}}$$

$$= \int \frac{du}{u^2}$$

$$= \int u^{-2}\, du$$

$$= -u^{-1} + C$$

$$= -\csc\theta + C$$

<u>Ex</u>: $\int \sin 3x \cos 7x\, dx$

$$= \int \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right] dx$$

$$= \frac{1}{2}\int \left[\sin(-4x) + \sin(10x)\right] dx$$

$$= \frac{1}{2}\left[\frac{-\cos(-4x)}{-4} - \frac{\cos(10x)}{10}\right] + C$$

$$= \frac{1}{2}\left[\frac{\cos(-4x)}{4} - \frac{\cos(10x)}{10}\right] + C$$

$$\text{OR } \frac{1}{2}\left[\frac{\cos 4x}{4} - \frac{\cos 10x}{10}\right] + C$$

8.4 Trig Substitution

Recall $\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

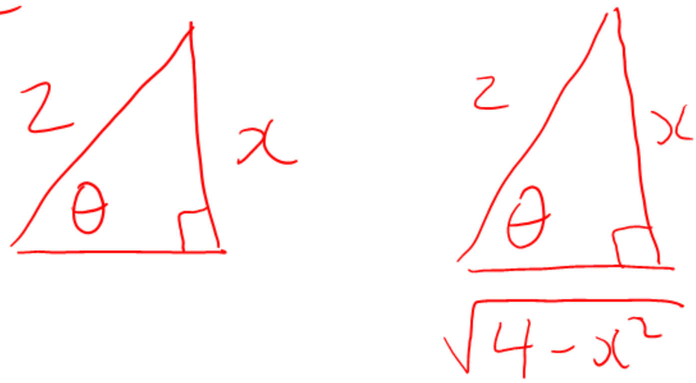
$\csc\theta = \frac{H}{O}$ $\sec\theta = \frac{H}{A}$ $\cot\theta = \frac{A}{O}$

If integral contains	then substitute
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$
$\sqrt{x^2 - a^2}$ $(a>0)$	$x = a\sec\theta$

Ex: Find $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$

$$x = 2\sin\theta$$
$$dx = 2\cos\theta\, d\theta$$

$$\frac{x}{2} = \sin\theta$$



$$\cos\theta = \frac{\sqrt{4-x^2}}{2}$$

$$2\cos\theta = \sqrt{4-x^2}$$

$$I = \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta\,(2\cos\theta)}$$

$$= \frac{1}{4}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{4}\cot\theta + C$$

$$= -\frac{1}{4}\frac{\sqrt{4-x^2}}{x} + C$$

<u>Ex</u>: Find $\displaystyle\int \frac{dx}{\sqrt{x^2+2}^{\,3}}$

$x = \sqrt{2}\tan\theta$

$\vdots$