Test 1

FRI SEPT 29

Bring calculator

Bring music/earplugs

Practice Problems on website

Covers : 1.2-1.5
2.2-2.4
5.1
2.5
4.4-4.5
5.2, 5.4
5.7
5.8
8.1

8.2 Integration By Parts Cont'd

Ex: $\int 2x^3 \cos x^2 \, dx$

$$\boxed{\begin{aligned} u &= x^2 \\ du &= 2x\,dx \end{aligned}}$$

$= \int 2x \cdot x^2 \cdot \cos x^2 \, dx$

$= \int u \cos u \, du$

	D	I
+	u	$\cos u$
−	1	$\sin u$
	0	$-\cos u$

$$= u\sin u + \cos u + C$$
$$= x^2 \sin x^2 + \cos x^2 + C$$

8.3 Trig Integrals

We'll use substitution to find:

$$\int \sin^n\theta \cos\theta \, d\theta$$

or $$\int \cos^n\theta \sin\theta \, d\theta$$

Fact $\sin^2\theta + \cos^2\theta = 1$

Ex: $\int \sin^4\theta \cos\theta \, d\theta$

$$\boxed{\begin{array}{l} u = \sin\theta \\ du = \cos\theta \, d\theta \end{array}}$$

$$= \int u^4 \, du$$
$$= \frac{u^5}{5} + C$$
$$= \frac{1}{5}\sin^5\theta + C$$

Ex: $\int \sin^4\theta \cos^3\theta \, d\theta$

$$= \int \sin^4\theta \, \underbrace{\cancel{\cos^2\theta}}_{(1-\sin^2\theta)} \cos\theta \, d\theta$$

$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\, d\theta \end{aligned}}$$

$$= \int u^4 (1-u^2)\, du$$

$$= \int (u^4 - u^6)\, du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C$$

$$= \frac{\sin^5\theta}{5} - \frac{\sin^7\theta}{7} + C$$

Ex: $\int \sin^5\theta\, d\theta$

$$= \int \sin^4\theta \underbrace{\sin\theta\, d\theta}$$

$$= \int \left(\sin^2\theta\right)^2 \sin\theta\, d\theta$$

$$= \int (1-\cos^2\theta)^2 \sin\theta\, d\theta$$

$$\boxed{\begin{aligned} u &= \cos\theta \\ du &= -\sin\theta\, d\theta \\ -du &= \sin\theta\, d\theta \end{aligned}}$$

$$= -\int (1-u^2)^2\, du$$

$$= -\int (1-2u^2+u^4)\, du$$

$$= -\left[u - \frac{2u^3}{3} + \frac{u^5}{5} \right] + C$$

$$= -\left[\cos\theta - \frac{2\cos^3\theta}{3} + \frac{\cos^5\theta}{5}\right] + C$$

To integrate even powers:

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Ex: $\int \cos^2\theta \, d\theta$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

Shortcuts

$$\int e^{4x} dx = \frac{e^{4x}}{4} + C$$

$$\int \cos 3x \, dx = \frac{\sin 3x}{3} + C$$

$$\int \sin 9x \, dx = \frac{-\cos 9x}{9} + C$$

Ex: $\int \sin^4 3x \, dx$

$$= \int (\sin^2 3x)^2 \, dx$$

$$= \int \left(\frac{1-\cos 6x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int \left[1 - 2\cos 6x + \cos^2 6x\right] dx$$

$$= \frac{1}{4}\int \left[1 - 2\cos 6x + \frac{1}{2} + \frac{\cos 12x}{2}\right] dx$$

$$= \frac{1}{4}\left[x - \frac{\sin 6x}{3} + \frac{x}{2} + \frac{\sin 12x}{24}\right] + C$$

To evaluate $\int \sec^m\theta \tan^n\theta \, d\theta$:

$$1 + \tan^2\theta = \sec^2\theta$$

Sub $u = \tan\theta$ $\quad du = \sec^2\theta \, d\theta$

OR

$u = \sec\theta$ $\quad du = \sec\theta\tan\theta \, d\theta$

$$\boxed{\begin{array}{l} \int \sec^2 x \, dx = \tan x + C \\ \frac{d}{dx}[\tan x] = \sec^2 x \end{array}}$$

Ex: $\int \tan^3\theta \sec^3\theta \, d\theta$

$$\int \boxed{\begin{matrix}\text{function}\\ \text{of } \tan\theta\end{matrix}} \sec^2\theta \, d\theta \quad \boxed{\text{OR}} \quad \int \boxed{\begin{matrix}\text{function}\\ \text{of } \sec\theta\end{matrix}} \sec\theta\tan\theta \, d\theta$$

$$\int \tan^3\theta \sec^3\theta \, d\theta$$

$$= \int \underset{(\sec^2\theta-1)}{\cancel{\tan^2\theta}} \sec^2\theta \quad \underbrace{\sec\theta\tan\theta \, d\theta}_{du}$$

$$\boxed{\begin{matrix} u = \sec\theta \\ du = \sec\theta\tan\theta \, d\theta \end{matrix}}$$

$$= \int (u^2-1)\, u^2 \, du$$

$$= \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C$$

<u>Ex</u>: $\int \tan^4\theta \, d\theta$

$$= \int \tan^2\theta \underset{(\sec^2\theta-1)}{\cancel{\tan^2\theta}} \, d\theta$$

$$= \int \left[\tan^2\theta\sec^2\theta - \underset{(\sec^2\theta-1)}{\cancel{\tan^2\theta}}\right] d\theta$$

$$= \int \left[\tan^2\theta \sec^2\theta - \sec^2\theta + 1\right] d\theta$$

$$= \int [\underbrace{\tan^2\theta \sec^2\theta - \sec^2\theta}_{(\tan^2\theta - 1)\sec^2\theta}] d\theta + \int 1 \, d\theta$$

$$\boxed{\begin{array}{l} u = \tan\theta \\ du = \sec^2\theta \, d\theta \end{array}}$$

$$= \int (u^2 - 1)\, du + \int d\theta$$

$$= \frac{u^3}{3} - u + \theta + C$$

$$= \frac{\tan^3\theta}{3} - \tan\theta + \theta + C$$

To evaluate $\int \csc^m\theta \cot^n\theta \, d\theta$:

$$1 + \cot^2\theta = \csc^2\theta$$

Sub $u = \cot\theta \qquad du = -\csc^2\theta \, d\theta$

OR

$u = \csc\theta \qquad du = -\csc\theta \cot\theta \, d\theta$

$$\boxed{\begin{aligned} &\int \csc^2 x\,dx = -\cot x + C \\ &\frac{d}{dx}[-\cot x] = \csc^2 x \\ &\frac{d}{dx}[\cot x] = -\csc^2 x \end{aligned}}$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

Ex: $\int \cot^2 3x \csc^4 3x\,dx$

$$= \int \cot^2 3x \underbrace{\cancel{\csc^2 3x}}_{(1+\cot^2 3x)} \underbrace{\csc^2 3x\,dx}$$

$$\boxed{\begin{aligned} u &= \cot 3x \\ du &= -3\csc^2 3x\,dx \\ -\tfrac{1}{3}\,du &= \csc^2 3x\,dx \end{aligned}}$$

$$= -\frac{1}{3}\int u^2(1+u^2)\,du$$

$$= -\frac{1}{3}\int (u^2+u^4)\,du$$

$$= -\frac{1}{3}\left[\frac{u^3}{3} + \frac{u^5}{5}\right] + C$$

$$= -\frac{1}{3}\left[\frac{\cot^3 3x}{3} + \frac{\cot^5 3x}{5}\right] + C$$