8.2 Integration By Parts Cont'd

$$\int u\,dv = uv - \int v\,du$$

Ex: $\int \arctan x\,dx$

$$\boxed{\begin{array}{ll} u = \arctan x & dv = 1\,dx \\ du = \dfrac{1}{1+x^2}\,dx & v = x \end{array}}$$

Don't write $+C$

$$\int u\,dv = uv - \int v\,du$$

$$\int \arctan x\,dx = x\arctan x - \int \frac{x}{1+x^2}\,dx$$

$$\boxed{\begin{aligned} w &= 1+x^2 \\ dw &= 2x\,dx \\ \tfrac{1}{2}\,dw &= x\,dx \end{aligned}}$$

$$= x\arctan x - \frac{1}{2}\int \frac{dw}{w}$$

$$= x\arctan x - \frac{1}{2}\ln|w| + C$$

$$= x\arctan x - \frac{1}{2}\ln|1+x^2| + C$$

Ex: $\int x\sqrt{1+x}\,dx$

$$\boxed{\begin{array}{ll} u = x & dv = (1+x)^{1/2}\,dx \\ du = dx & v = \frac{2}{3}(1+x)^{3/2} \end{array}}$$

$$\int x\sqrt{1+x}\,dx = uv - \int v\,du$$

$$= \frac{2}{3}x(1+x)^{3/2} - \int \frac{2}{3}(1+x)^{3/2}\,dx$$

$$= \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(1+x)^{5/2} + C$$

$$= \frac{2}{3}x(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} + C$$

Ex: $\int e^{2x}\cos x\,dx$

$$\boxed{\begin{array}{ll} u = e^{2x} & dv = \cos x\,dx \\ du = 2e^{2x}\,dx & v = \sin x \end{array}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\underbrace{\int e^{2x}\sin x\,dx}_{\text{Integration By Parts}} \qquad 1$$

$$\boxed{\begin{array}{ll} u = e^{2x} & dv = \sin x\,dx \\ du = 2e^{2x}\,dx & v = -\cos x \end{array}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx \qquad 2$$

2 → 1 :

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\left[-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx\right]$$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x\,dx$$

Let $I = \int e^{2x} \cos x \, dx$

$$I = e^{2x} \sin x + 2e^{2x} \cos x - 4I$$

$$5I = e^{2x} \sin x + 2e^{2x} \cos x + C$$

$$I = \frac{1}{5} e^{2x} (\sin x + 2\cos x) + C_1$$

Tabular Method

Can be useful when the integral contains x^n.

Ex: $\int x^2 \cos 3x \, dx$

	D	I
$+$	x^2	$\cos 3x$
$-$	$2x$	$\frac{1}{3} \sin 3x$
$+$	2	$-\frac{1}{9} \cos 3x$
	0	$-\frac{1}{27} \sin 3x$

$$\int x^2 \cos 3x \, dx = \frac{x^2}{3}\sin 3x + \frac{2x}{9}\cos 3x - \frac{2}{27}\sin 3x + C$$

<u>Ex</u>: $\int (x^3+2x)e^{2x} \, dx$

	D	I
$+$	(x^3+2x)	e^{2x}
$-$	$(3x^2+2)$	$e^{2x}/2$
$+$	$6x$	$e^{2x}/4$
$-$	6	$e^{2x}/8$
	0	$e^{2x}/16$

$$\int (x^3+2x)e^{2x}\,dx = \frac{1}{2}(x^3+2x)e^{2x}$$

$$-\frac{1}{4}(3x^2+2)e^{2x} + \frac{6x}{8}e^{2x}$$

$$-\frac{6}{16}e^{2x} + C$$