Recap: Derivatives

$f(x)$	$f'(x)$
$\frac{1}{x^2} = x^{-2}$	$-2x^{-3}$
$3\sqrt{x} = 3x^{1/2}$	$\frac{3}{2}x^{-1/2}$
$\ln g(x)$	$\frac{1}{g(x)} \cdot g'(x)$
$e^{g(x)}$	$g'(x)\, e^{g(x)}$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$

__Ex:__ $y = \cos^3(8x)$

Find y'

$y = [\cos(8x)]^3$

$y' = 3[\cos(8x)]^2[-\sin(8x)\cdot 8]$

$= -24\sin 8x \cos^2 8x$

Can't find $\int \sqrt{\sin x}\,dx$, at least not easily.

__Ex:__ $\int \sqrt{\sin x}\cos x\,dx$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \end{aligned}}$$

$= \int \sqrt{u}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(\sin x)^{3/2} + C$

Ex: $\displaystyle\int \frac{6\,dx}{(\sin^{-1}x)\sqrt{1-x^2}}$

$$u = \sin^{-1}x$$
$$du = \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= 6\int \frac{du}{u}$$
$$= 6\ln|u| + C$$
$$= 6\ln|\sin^{-1}x| + C$$

Ex: $\displaystyle\int \frac{e^{-1/x}\,dx}{x^2}$

$$u = -\frac{1}{x}$$
$$du = x^{-2}\,dx$$

$$= \int e^u\,du$$
$$= e^u + C$$
$$= e^{-\frac{1}{x}} + C$$

5.7 Derivatives of Inverse Trig Functions

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$$

Ex: Find y'

c) $y = 2x\arctan x - \ln(1+x^2)$

$$y' = 2x \cdot \frac{1}{1+x^2} + \arctan x\,(2) - \frac{1}{1+x^2}(2x)$$

$$= 2\arctan x$$

d) $y = 12\arcsin\frac{x}{4}$

$$y' = 12\,\frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}}\left(\frac{1}{4}\right)$$

$$= \frac{12}{\sqrt{1-\frac{x^2}{16}}\,\sqrt{16}}$$

$$= \frac{12}{\sqrt{16 - x^2}}$$

<u>Ex</u>: Prove that $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

$$\sin(\arcsin x) = x$$

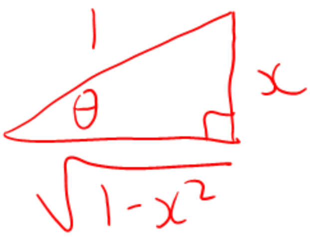
Take $\frac{d}{dx}$: $\cos(\arcsin x) \frac{d}{dx} \arcsin x = 1$

$$\frac{d}{dx} \arcsin x = \frac{1}{\cos(\arcsin x)}$$

Let $\theta = \arcsin x$

$$\sin\theta = x$$

$$\sin\theta = \frac{x}{1}$$



$$\begin{cases} a^2 + b^2 = c^2 \\ a^2 + x^2 = 1 \\ a^2 = 1 - x^2 \\ a = \sqrt{1-x^2} \end{cases}$$

$$\cos\theta = \sqrt{1-x^2}$$

$$\frac{d}{dx} \arcsin x = \frac{1}{\cos(\arcsin x)}$$

$$= \frac{1}{\cos \theta}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

5.8 Inverse Trig Functions: Integration

Let $a > 0$.

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin \frac{x}{a} + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

Ex: Find $\int \frac{1}{\sqrt{16-x^2}}\,dx$

$$= \arcsin \frac{x}{4} + C$$

Ex: Find $\int \frac{1}{9+4x^2}\,dx$

$$= \int \frac{dx}{3^2 + (2x)^2}$$

$$\boxed{\begin{aligned} u &= 2x \\ du &= 2\,dx \\ \frac{du}{2} &= dx \end{aligned}}$$

$$= \frac{1}{2} \int \frac{du}{3^2 + u^2}$$

$$= \frac{1}{2}\left(\frac{1}{3} \arctan \frac{u}{3}\right) + C$$

$$= \frac{1}{6} \arctan \frac{2x}{3} + C$$

<u>Ex</u>: Find $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{1+\sin^2\theta}\, d\theta$

$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\, d\theta \\ \theta = 0 &\Rightarrow u = 0 \\ \theta = \tfrac{\pi}{2} &\Rightarrow u = 1 \end{aligned}}$$

$$= \int_0^1 \frac{du}{1+u^2}$$

$$= \arctan u \Big|_0^1$$

$$= \frac{\pi}{4} - 0$$
$$= \frac{\pi}{4}$$

Complete the Square

$x^2 + 10x + 34$

$$\boxed{\begin{array}{l} \frac{10}{2} = 5 \\ 5^2 = 25 \end{array}}$$

$$= x^2 + 10x + 25 + \cancel{34 - 25}^{\,9}$$
$$= (x+5)^2 + 3^2$$

Ex: $\displaystyle\int \frac{dx}{x^2 + 6x + 13}$

$$\boxed{\begin{array}{l} x^2 + 6x + 13 \\ = x^2 + 6x + 9 + \cancel{13 - 9}^{\,4} \\ = (x+3)^2 + 2^2 \end{array}}$$

$$= \int \frac{dx}{(x+3)^2 + 2^2}$$

$$\boxed{\begin{array}{l} u = x + 3 \\ du = dx \end{array}}$$

$$= \int \frac{du}{2^2 + u^2}$$

$$= \frac{1}{2}\arctan\frac{u}{2} + C$$

$$= \frac{1}{2}\arctan\frac{x+3}{2} + C$$

<u>Ex</u>: $\displaystyle\int \frac{dx}{\sqrt{8x-x^2}}$

$$\boxed{\begin{aligned} &8x - x^2 \\ &= -x^2 + 8x \\ &= -(x^2 - 8x) \\ &= -(x^2 - 8x + 16) + 16 \\ &= 4^2 - (x-4)^2 \end{aligned}}$$

$$= \int \frac{dx}{\sqrt{4^2-(x-4)^2}}$$

$$\boxed{\begin{aligned} u &= x-4 \\ du &= dx \end{aligned}}$$

$$= \int \frac{du}{\sqrt{4^2-u^2}}$$

$$= \sin^{-1}\frac{u}{4} + C$$

$$= \sin^{-1}\frac{x-4}{4} + C$$