Do Sugg. HW for
Sections 11.7, 12.2, 12.4

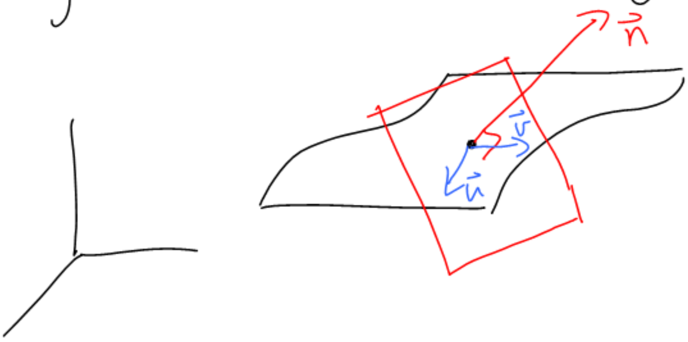
Notation in Sugg. HW

exp(x) means e^x

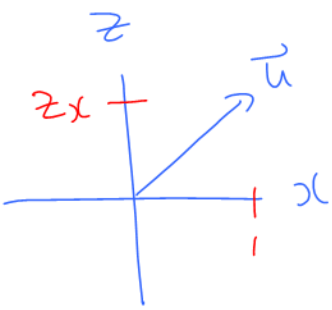
exp(x^2+2) " e^{x^2+2}

12.4 Cont'd

Q: why does $\vec{n} = [-z_x, -z_y, 1]$?



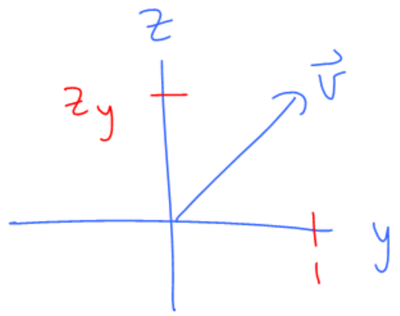
$\vec{u}, \vec{v}$: direction vectors for the tangent plane parallel to the x-axis and y-axis



slope of $\vec{u}$ = rate of change of z in the x-direction

$= z_x$

$\vec{u} = [1, 0, z_x]$



slope of $\vec{v}$ = rate of change of z in y-direction

$$= z_y$$

$$\vec{v} = [0, 1, z_y]$$

$$\vec{n} = \vec{u} \times \vec{v}$$

$$\begin{array}{ccccc} 1 & 0 & z_x & 1 & 0 \\ 0 & 1 & z_y & 0 & 1 \end{array}$$

$$\vec{n} = [-z_x, -z_y, 1]$$

<u>Ex</u>: Find all points (x, y, z) where the tangent plane is horizontal:

$$z = x^3 - 12x + y^2 + 8y + 10$$

$$z_x = 3x^2 - 12$$

$$z_y = 2y + 8$$

Set $z_x = 0$ and $z_y = 0$

$$3x^2-12=0$$
$$3(x^2-4)=0$$
$$3(x-2)(x+2)=0$$
$$x=\pm 2$$

$$2y+8=0$$
$$2y=-8$$
$$y=-4$$

Original equation

$$(x,y,z) = (2,-4,-22)$$
$$(x,y,z) = (-2,-4,10)$$

Ex: Show that $T = e^{-k\omega^2 t}\sin(\omega x)$ satisfies $\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}$ (where k, w are constants)

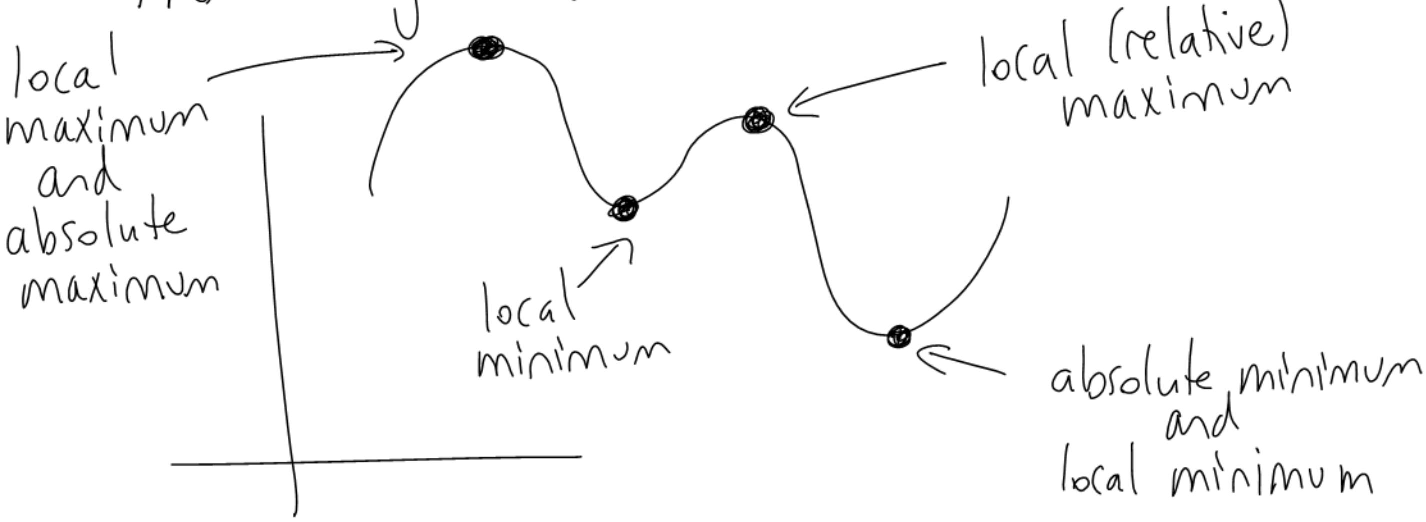
We want to confirm $T_t = k T_{xx}$

$$T_t = -k\omega^2 e^{-k\omega^2 t}\sin(\omega x)$$
$$T_x = \omega e^{-k\omega^2 t}\cos(\omega x)$$
$$T_{xx} = -\omega^2 e^{-k\omega^2 t}\sin(\omega x)$$

$$kT_{xx} = -k\omega^2 e^{-k\omega^2 t}\sin(\omega x)$$
$$= T_t \quad \checkmark$$

12.5 Multivariable Optimization

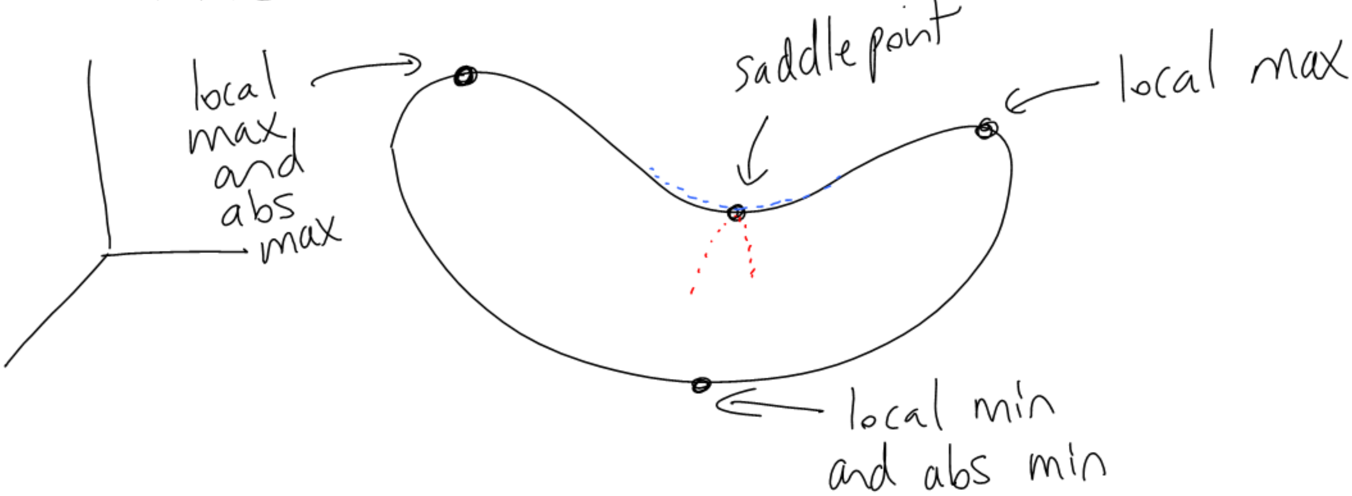
From single-variable calculus:



Def

> Points where z_x and z_y are both 0 or undefined are called <u>critical points</u>.

1) Could have $z_x = 0$ and z_y is undefined or vice versa.
2) Critical points may be a local max, a local min, a saddle point, or none of these.



Ex: Find all critical points of

$$z = x^3 + xy^2 + 3x^2 - 3y^2$$

$z_x = 3x^2 + y^2 + 6x \qquad z_y = 2xy - 6y$

$\underbrace{\qquad\qquad\qquad\qquad\qquad}_{\text{both } 0 \text{ or undefined}}$

$$3x^2 + y^2 + 6x = 0 \quad ①$$
$$2xy - 6y = 0 \quad ②$$

②: $2xy - 6y = 0$

$2y(x-3) = 0$

$2y = 0 \qquad x - 3 = 0$

$y = 0$ OR $x = 3$

Case 1: $y = 0$

$y = 0 \Rightarrow ①$

$3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0, -2$

$\boxed{(0,0), (-2,0)}$

Case 2: $x = 3$

$x = 3 \Rightarrow ①$

$27 + y^2 + 18 = 0$

$y^2 = -45$

no solution

Critical Points: $(0,0), (-2,0)$

FACT

region of xy-plane that includes its boundary

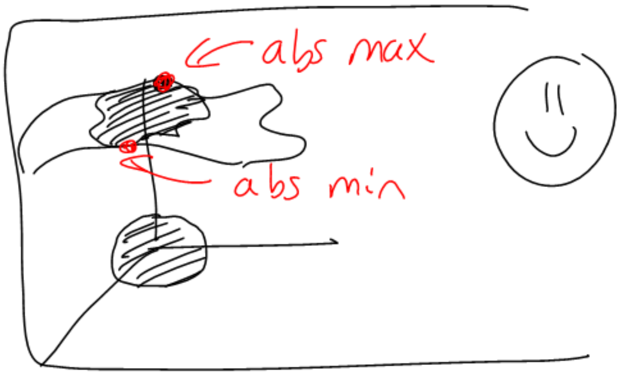
A continuous function defined over a closed domain attains an absolute maximum and an absolute minimum.

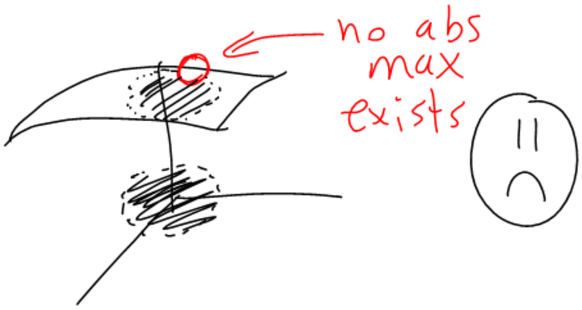
closed domain

not a closed domain

closed domain

continuous surface





FACT

The absolute maximum and absolute minimum of f may occur in the interior of the domain, at a corner of the domain, or on a side of the domain.

Ex: Find the absolute maximum and absolute minimum of $f = x + x^2 + y^2$ over the region below: