2.5 Implicit Differentiation Cont'd

Ex: y depends on x

a) $\frac{d}{dx}[x^3] = 3x^2$

b) $\frac{d}{dx}[y^3] = 3y^2 \frac{dy}{dx}$

c) $\frac{d}{dx}[(7x^2)y^4]$

$= 7x^2\left(4y^3\frac{dy}{dx}\right) + y^4(14x)$

$= 28x^2y^3\frac{dy}{dx} + 14xy^4$

Ex: Find the tangent line to $x^3 + y^3 = 9xy$ at $(2,4)$.

$(9x)y$

Need $\frac{dy}{dx}$

1) Take $\frac{d}{dx}$

$3x^2 + 3y^2\frac{dy}{dx} = 9x\frac{dy}{dx} + y(9)$

2) Solve for $\frac{dy}{dx}$

$$3y^2\frac{dy}{dx} - 9x\frac{dy}{dx} = 9y - 3x^2$$

$$\left[3y^2 - 9x\right]\frac{dy}{dx} = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x}$$

$$\left.\frac{dy}{dx}\right|_{(2,4)} = \frac{24}{30} = \frac{4}{5}$$

$m = \frac{4}{5}$ $\quad x_1 = 2$ $\quad y_1 = 4$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{4}{5}(x - 2) \checkmark$$

$$y - 4 = \frac{4}{5}x - \frac{8}{5} \checkmark$$

$$y = \frac{4}{5}x + \frac{12}{5} \checkmark$$

Ex: Find $\frac{dy}{dx}$ given

$$x^4 y + y^4 = 1 + \sin(xy)$$

$$x^4 \frac{dy}{dx} + y(4x^3) + 4y^3 \frac{dy}{dx} = \cos(xy)\left[x\frac{dy}{dx} + y(1)\right]$$

$$x^4 \frac{dy}{dx} + 4x^3 y + 4y^3 \frac{dy}{dx} = x\cos(xy)\frac{dy}{dx} + y\cos(xy)$$

$$x^4 \frac{dy}{dx} + 4y^3 \frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = -4x^3 y + y\cos(xy)$$

$$\left[x^4 + 4y^3 - x\cos(xy)\right]\frac{dy}{dx} = y\cos(xy) - 4x^3 y$$

$$\frac{dy}{dx} = \frac{y\cos(xy) - 4x^3 y}{x^4 + 4y^3 - x\cos(xy)}$$

4.4-4.5 Review of Integration

The indefinite integral $\int f(x)\,dx$ represents all antiderivatives of $f(x)$.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

Ex: Find:

a) $\int (x^5 + 2x^4 - 5x + 3)\,dx$

$= \frac{x^6}{6} + \frac{2x^5}{5} - \frac{5x^2}{2} + 3x + C$

b) $\int \left(\sqrt{x} + \frac{1}{x^3}\right) dx$

$= \int (x^{1/2} + x^{-3})\,dx$

$= \frac{2}{3}x^{3/2} - \frac{1}{2}x^{-2} + C$

or $\frac{2x^{3/2}}{3} - \frac{x^{-2}}{2} + C$

Ex: Find

a) $\int 5x^2 (x^3+1)^6\,dx$

$$\boxed{\begin{aligned} &\text{Let } u = x^3+1 \\ &\frac{du}{dx} = 3x^2 \\ &du = 3x^2\,dx \\ &\frac{du}{3} = x^2\,dx \end{aligned}}$$

$$= \frac{5}{3}\int u^6 \, du$$

$$= \frac{5}{3}\left(\frac{u^7}{7}\right) + C$$

$$= \frac{5}{21}\left(x^3+1\right)^7 + C$$

b) $$\int \frac{x}{\sqrt{2x^2+1}} \, dx$$

$$u = 2x^2+1$$
$$\frac{du}{dx} = 4x$$
$$du = 4x\,dx$$
$$\frac{du}{4} = x\,dx$$

$$= \frac{1}{4}\int \frac{du}{\sqrt{u}}$$

$$= \frac{1}{4}\int u^{-1/2} \, du$$

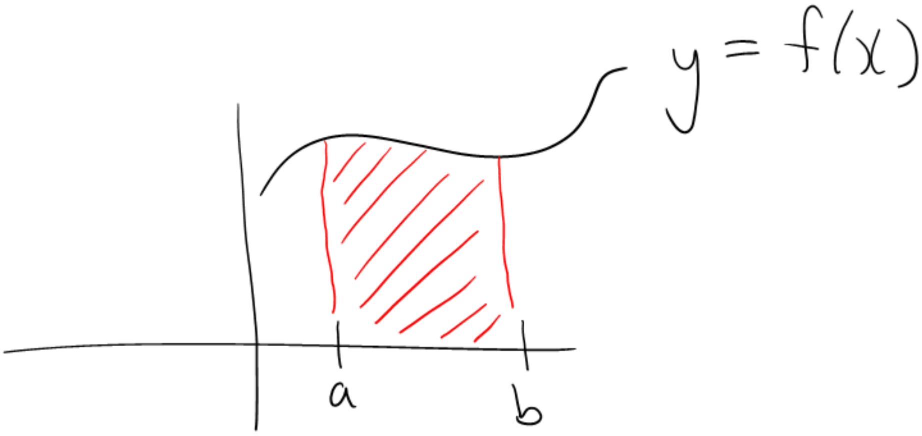
$$= \frac{1}{4}\left(2u^{1/2}\right) + C$$

$$= \frac{1}{2}\sqrt{2x^2+1} + C$$

The definite integral $\int_a^b f(x)\,dx$

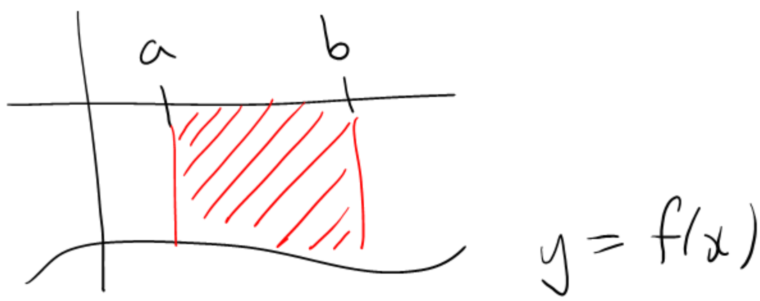
represents area under a curve,

if $f(x) \geq 0$.



$\int_a^b f(x)\,dx < 0$

when the curve is below the x-axis.



$$\int_a^b f(x)\,dx = \mathbf{F}(b) - \mathbf{F}(a)$$

where $\mathbf{F}(x)$ is an antiderivative of $f(x)$

$$\int_1^4 x^2\,dx = \left.\frac{x^3}{3}\right|_1^4$$

$$= \frac{4^3}{3} - \frac{1^3}{3}$$

$$= \frac{63}{3}$$

$$= 21$$