Test 4 # 3

Rewrite as a power series centred at $c = -1$

$$f(x) = \frac{2}{5x+4}$$

$$= \frac{2}{5(x+1) + ?}$$

$$= \frac{2}{5(x+1) - 1}$$

$$= \frac{-2}{-5(x+1) + 1}$$

$$= -2 \cdot \frac{1}{1 - 5(x+1)}$$

$$= -2 \sum_{n=0}^{\infty} [5(x+1)]^n \checkmark$$

$$= -2 \sum_{n=0}^{\infty} 5^n (x+1)^n \checkmark$$

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$$(1+x)^k \approx 1 + kx + \tfrac{1}{2}k(k-1)x^2$$

$$(1+x)^{1/3} \approx 1 + \tfrac{1}{3}x + \tfrac{1}{2}\left(\tfrac{1}{3}\right)\left(-\tfrac{2}{3}\right)x^2$$

$$\approx 1 + \frac{x}{3} - \frac{x^2}{9}$$

$$(1+x^2)^{1/3} \approx 1 + \frac{x^2}{3} - \frac{x^4}{9}$$

$$\int_0^{0.5} \sqrt[3]{1+x^2}\,dx \approx \int_0^{0.5} \left(1 + \frac{x^2}{3} - \frac{x^4}{9}\right)dx$$

$$\approx \left[x + \frac{x^3}{9} - \frac{x^5}{45}\right]_0^{0.5}$$

$$\approx 0.5132$$

(32) $\vec{a}(t) = [4, 0, -9.8]$

$$\vec{v}(t) = \int \vec{a}(t)\,dt$$

$$= \int [4, 0, -9.8]\,dt$$

$$= [4t, 0, -9.8t] + \vec{c}_1$$

Sub $t=0$:

$$[0, 25, 25] = \vec{0} + \vec{c}_1$$

$$\vec{c}_1 = [0, 25, 25]$$

$$\boxed{\vec{v}(t) = [4t, 25, -9.8t + 25]}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt$$

$$= [2t^2, 25t, -4.9t^2 + 25t] + \vec{c}_2$$

Sub $t=0$:

$$[0, 0, 0] = \vec{0} + \vec{c}_2$$

$$\vec{c}_2 = \vec{0}$$

$$\boxed{\vec{r}(t) = [2t^2, 25t, -4.9t^2 + 25t]}$$

It hits the ground when

$(z\text{-component of } \vec{r}(t)) = 0$

$$-4.9t^2 + 25t = 0$$

$$t(-4.9t + 25) = 0$$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$t = 0 \qquad -4.9t + 25 = 0$

$$t = \frac{25}{4.9}$$

$$\vec{v}\left(\frac{25}{4.9}\right) = \left[\frac{100}{4.9}, 25, -25\right]$$

$$\left\|\vec{v}\left(\frac{25}{4.9}\right)\right\| = \sqrt{\left(\frac{100}{4.9}\right)^2 + (25)^2 + (-25)^2}$$

$$\simeq 41 \text{ m/s}$$

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$$a_T = \frac{\vec{v}\cdot\vec{a}}{\|\vec{v}\|} \qquad a_N = \frac{\|\vec{v}\times\vec{a}\|}{\|\vec{v}\|}$$

$$\vec{r} = \left[t, 1 + \frac{1}{t}, 0\right]$$

$$\vec{v} = [1, -t^{-2}, 0]$$

$$\vec{a} = [0, 2t^{-3}, 0]$$

$$\vec{v}\cdot\vec{a} = 1(0) + (-t^{-2})(2t^{-3}) + 0(0)$$

$$= -2t^{-5}$$

$$\|\vec{v}\| = \sqrt{1^2 + (-t^{-2})^2 + 0^2}$$

$$= \sqrt{1 + t^{-4}}$$

$$a_T = \frac{-2t^{-5}}{\sqrt{1+t^{-4}}}$$

$$\vec{v}\times\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -t^{-2} & 0 \\ 0 & 2t^{-3} & 0 \end{vmatrix}$$

$$= \vec{i}(0) - \vec{j}(0) + \vec{k}(2t^{-3})$$

$$= [0, 0, 2t^{-3}]$$

$$\|\vec{r} \times \vec{a}\| = \sqrt{0^2 + 0^2 + (2t^{-3})^2}$$

$$= \sqrt{4t^{-6}}$$

$$= |2t^{-3}|$$

$$= 2t^{-3} \qquad (t>0)$$

$$a_N = \frac{2t^{-3}}{\sqrt{1+t^{-4}}}$$

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$$S = \int \underbrace{\|\vec{v}\|}_{\text{speed}} \, dt$$

$$\vec{r} = [\cos^3 t, \sin^3 t]$$

$$\vec{v} = [3\cos^2 t(-\sin t), 3\sin^2 t \cos t]$$

$$\|\vec{v}\| = \sqrt{[-3\cos^2 t \sin t]^2 + [3\sin^2 t \cos t]^2}$$

$$= \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}$$

$$= \sqrt{9\cos^2 t \sin^2 t \, (\cancel{\cos^2 t + \sin^2 t})}$$

$$= |3\cos t \sin t|$$

$$= 3\cos t\sin t \qquad \left(0 \le t \le \frac{\pi}{2}\right)$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$= \frac{3}{2}\sin 2t$$

$$s = \int_0^{\pi/2} \frac{3}{2}\sin 2t \, dt$$

$$= -\frac{3}{4}\cos 2t \Big|_0^{\pi/2}$$

$$= -\frac{3}{4}(-1) + \frac{3}{4}$$

$$= \frac{3}{2}$$

ASIDE $\quad \vec{r}(t) = [\cos^3 t, \sin^3 t]$

$$\begin{cases} x = \cos^3 t \\ y = \sin^3 t \end{cases}$$

speed

$$s = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$\vdots$$

$$s = \frac{3}{2}$$