FINAL EXAM

Thurs Dec 14

1:30-4:30pm

TEC 175

No Music Allowed

Bring Calculator, Earplugs

$\boxed{\text{14 Questions}}$

Sections	% of Marks on Exam
8.2-8.5, 5.6, 8.8	30
9.1-9.10	28
10.2-10.5	25
12.1-12.5	17

Review Problems on website.

21 g) Let $a_n = \dfrac{n!}{1(3)(5)\cdots(2n-1)}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\overset{(n+1)}{\cancel{(n+1)!}}}{\cancel{1(3)\cdots}(2n+1)}\cdot\frac{\cancel{1(3)\cdots(2n-1)}}{\cancel{n!}}\right|$$

$$= \lim_{n\to\infty}\frac{n+1}{2n+1} \leftarrow \frac{\infty}{\infty}$$

$$\overset{H}{=} \lim_{n\to\infty}\frac{1}{2}$$

$$= \frac{1}{2}$$

The series converges by the Ratio Test.

h) Root Test

$$a_n = \frac{1}{(1+\ln n)^n}$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|\frac{1}{(1+\ln n)^n}\right|}$$

$$= \lim_{n\to\infty} \sqrt[n]{\frac{1}{(1+\ln n)^n}}$$

$$= \lim_{n\to\infty} \frac{1}{1+\ln n}$$

$$= 0$$

Series converges by the Root Test.

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$$|R_N(x)| = \left| \frac{f^{(N+1)}(z)}{(N+1)!} (x-c)^{N+1} \right|$$

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f^{(N+1)}(x) = e^x \qquad f^{(N+1)}(z) = e^z$$

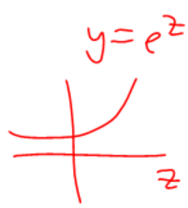
$$x = -0.2$$

$c = 0$ (Maclaurin polynomial)

$$|R_N(-0.2)| = \left| \frac{e^z}{(N+1)!} (-0.2)^{N+1} \right|$$

z is between $\cancel{x}$ and $\cancel{c}$
$\quad -0.2 \qquad 0$

e^z is increasing on $-0.2 \le z \le 0$



$$e^z \le \cancel{e^0} \quad 1$$

$$|R_N(-0.2)| \le \left| \frac{1}{(N+1)!} (-0.2)^{N+1} \right|$$

$$\le \frac{0.2^{N+1}}{(N+1)!}$$

N	$\frac{0.2^{N+1}}{(N+1)!} < 0.0001$?
1	No
2	No
3	YES

$$\boxed{N \geqslant 3}$$

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$$a_n = \frac{(x-1)^n}{n^2 \cdot 2^n}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(x-1)^{n+1}}(x-1)}{(n+1)^2 \cdot \cancel{2^{n+1}}\, 2} \cdot \frac{n^2 \cdot \cancel{2^n}}{\cancel{(x-1)^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x-1}{2} \cdot \frac{n^2}{(n+1)^2} \right|$$

$$\frac{\infty}{\infty}$$

$$\stackrel{\text{H}}{=} \lim_{n\to\infty} \left| \frac{x-1}{2} \cdot \frac{2n}{2(n+1)} \right|$$

$$\stackrel{\text{H}}{=} \lim_{n\to\infty} \left| \frac{x-1}{2} \cdot \frac{\cancel{2}}{\cancel{2}} \right|$$

$$= \left| \frac{x-1}{2} \right|$$

Series converges if $\left| \frac{x-1}{2} \right| < 1$

$$|x-1| < 2$$

$$-2 < x-1 < 2$$

$$-1 < x < 3$$

$x = -1$:

$$\text{Series} = \sum_{n=1}^{\infty} \frac{\cancel{(-2)^n}\ (-1)^n \cancel{2^n}}{n^2 \cdot \cancel{2^n}}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

Series converges by Alternating Series Test

$x = 3$:

$$\text{Series} = \sum_{n=1}^{\infty} \frac{\cancel{2^n}\ 1}{n^2 \cdot \cancel{2^n}}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^2}$$

Series converges (p-series with $p=2$)

$$\boxed{-1 \leq x \leq 3}$$

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Use formula sheet

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

$$e^{-2x} \approx 1 - 2x + \frac{(-2x)^2}{2} + \frac{(-2x)^3}{6}$$

$$\approx 1 - 2x + 2x^2 - \frac{4x^3}{3}$$

$$xe^{-2x} \approx x - 2x^2 + 2x^3 - \frac{4x^4}{3}$$