(28)

$$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = 1-\cos t$$

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{\underline{\underline{\sin^2 t}} + 1 - 2\cos t + \underline{\underline{\cos^2 t}}}\, dt$$

$$= \int_0^{\pi} \sqrt{2 - 2\cos t}\, dt$$

$$\sin^2\theta = \frac{1}{2} - \frac{\cos 2\theta}{2}$$

$$\frac{1}{2} - \frac{\cos 2\theta}{2} = \sin^2\theta$$

$$2 - 2\cos 2\theta = 4\sin^2\theta$$

$$2 - 2\cos t = 4\sin^2 \frac{t}{2}$$

$$= \int_0^{\pi} \sqrt{4\sin^2 \frac{t}{2}}\, dt$$

$$= \int_0^{\pi} \left| 2\sin\frac{t}{2} \right| dt$$

$$= \int_0^{\pi} 2\sin\frac{t}{2}\, dt$$

$2\sin\frac{t}{2} \geq 0$ on $0 \leq t \leq \pi$

$$= -4\cos\frac{t}{2}\Big|_0^{\pi}$$

$$= 0 - (-4)$$

$$= 4$$

31 a)

$$\frac{d}{dt}\, 9t\,[t^2, t^3]$$

$$= \frac{d}{dt}\,[9t^3, 9t^4]$$

$$= [27t^2, 36t^3]$$

$$\frac{d}{dt}\, 9t\,[t^2, t^3]$$

$$= 9t\,[2t, 3t^2] + 9\,[t^2, t^3]$$

$$= [18t^2, 27t^3] + [9t^2, 9t^3]$$

$$= [27t^2, 36t^3]$$ ✓

b)

$$\frac{d}{dt}\,\vec{r}(2t)$$

$$= \frac{d}{dt}\,[14t+1, 8t]$$

$$= [14, 8]$$

$$\frac{d}{dt}\,\vec{r}(2t)$$

$$= \vec{r}\,'(2t)\ (2)$$

$$\boxed{\begin{array}{l}\vec{r}(t) = [7t+1, 4t] \\ \vec{r}\,'(t) = [7, 4]\end{array}}$$

$$= [7, 4]\ (2)$$

$$= [14, 8] \checkmark$$

d) $\int_1^3 [6t^2, 8t]\,dt$

$$= [2t^3, 4t^2]\Big|_1^3$$

$$= [54, 36] - [2, 4]$$

$$= [52, 32]$$

ASIDE

$$\int [6t^2, 8t]\,dt$$

$$= [2t^3, 4t^2] + \vec{C} \checkmark$$

$$\text{or } [2t^3 + C_1, 4t^2 + C_2] \checkmark$$

11

$u = \ln x \qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{x^3}{3}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \ln x\,dx = \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,dx$$

$$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$

(12) $\int \sec^4\theta \tan^3\theta \, d\theta$

$$= \int (\text{powers of } \tan\theta) \underbrace{\sec^2\theta \, d\theta}_{du}$$

$$= \int \underset{(1+\tan^2\theta)}{\cancel{\sec^2\theta}} \tan^3\theta \underbrace{\sec^2\theta \, d\theta}_{du}$$

$$= \int (\tan^3\theta + \tan^5\theta) \sec^2\theta \, d\theta$$

$$\boxed{\begin{aligned} u &= \tan\theta \\ du &= \sec^2\theta \, d\theta \end{aligned}}$$

$$= \int (u^3 + u^5) \, du$$

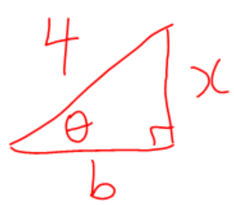
$$= \frac{u^4}{4} + \frac{u^6}{6} + C$$

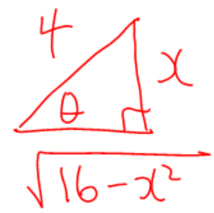
$$= \frac{\tan^4\theta}{4} + \frac{\tan^6\theta}{6} + C$$

(13) Sub $x = 4\sin\theta$

$dx = 4\cos\theta \, d\theta$

$$\frac{x}{4} = \sin\theta$$





$b^2 + x^2 = 4^2$

⋮

$$\boxed{\begin{aligned}\frac{\sqrt{16-x^2}}{4} &= \cos\theta \\ \sqrt{16-x^2} &= 4\cos\theta\end{aligned}}$$

$$\text{Integral} = \int \frac{dx}{x^2\sqrt{16-x^2}}$$

$$= \int \frac{\cancel{4\cos\theta}\, d\theta}{(4\sin\theta)^2\,(\cancel{4\cos\theta})}$$

$$= \frac{1}{16}\int \csc^2\theta \, d\theta$$

$$= -\frac{1}{16}\cot\theta + C$$

$$= -\frac{1}{16}\,\frac{\sqrt{16-x^2}}{x} + C$$

14

$$\frac{13}{(x+2)(x^2+9)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+9}$$

$$13 = A(x^2+9) + (Bx+C)(x+2)$$

$x=-2$: $\quad 13 = 13A \implies A = 1$

x^2 coefficient: $\quad 0 = A + B \Rightarrow B = -1$

$x = 0$: $\quad 13 = 9A + C(2)$

$$13 = 9 + 2C$$

$$C = 2$$

$$\text{Integral} = \int \left[\frac{1}{x+2} + \frac{-x+2}{x^2+9} \right] dx$$

$$= \int \left[\frac{1}{x+2} - \frac{x}{x^2+9} + \frac{2}{x^2+9} \right] dx$$

$$= \ln|x+2| - \frac{1}{2}\ln|x^2+9| + \frac{2}{3}\tan^{-1}\frac{x}{3} + C$$

15 a) $\displaystyle \lim_{x \to 0} \frac{\sin 6x}{\tan 7x} \leftarrow \frac{0}{0}$

$$\overset{H}{=} \lim_{x \to 0} \frac{6\cos 6x}{7\sec^2 7x}$$

$$= \frac{6}{7}$$

b) $\displaystyle \lim_{x \to 0^+} (e^x + 5x)^{1/x} \leftarrow 1^\infty$

Let $\displaystyle L = \lim_{x \to 0^+} (e^x + 5x)^{1/x}$

$$\ln L = \lim_{x \to 0^+} \ln (e^x + 5x)^{1/x}$$

$$= \lim_{x \to 0^+} \frac{\ln (e^x + 5x)}{x} \leftarrow \frac{0}{0}$$

$$\overset{H}{=} \lim_{x \to 0^+} \frac{\frac{1}{e^x + 5x}(e^x + 5)}{1}$$

$$= \frac{\frac{1}{1}(6)}{1}$$

$$= 6$$

$$\ln L = 6 \implies L = e^{\ln L} = e^6$$