Test Review

Ex: Let $\vec{r}(t) = [7t - \cos 2t, 3t^2, e^{-t}]$

Find $\vec{r}'(0) \cdot \vec{r}''(0)$ and $\vec{r}'(0) \times \vec{r}''(0)$

$$\vec{r}'(t) = [7 + 2\sin 2t, 6t, -e^{-t}]$$
$$\vec{r}''(t) = [4\cos 2t, 6, e^{-t}]$$

$$\vec{r}'(0) = [7, 0, -1]$$
$$\vec{r}''(0) = [4, 6, 1]$$

$$\vec{r}'(0) \cdot \vec{r}''(0) = 28 + 0 - 1$$
$$= 27$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 0 & -1 \\ 4 & 6 & 1 \end{vmatrix}$$

$$= \vec{i}(6) - \vec{j}(11) + \vec{k}(42)$$
$$= [6, -11, 42]$$

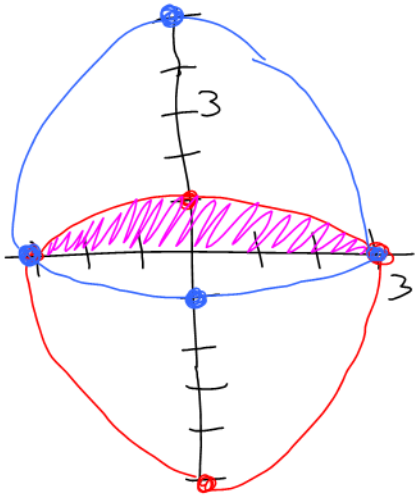
$$\vec{r}'(0) \times \vec{r}''(0) = [6, -11, 42]$$

Exam Review Problems

Problems + Solutions on website

Questions 1-10 are review material from beginning of the course

(30)



$r = 3 - 2\sin\theta$

$r = -3 + 2\sin\theta$

A = double the shaded area

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 \, d\theta$$

$$= 2 \cdot \frac{1}{2}\int_{0}^{\pi} (3 - 2\sin\theta)^2 \, d\theta$$

$$= \int_{0}^{\pi} (9 - 12\sin\theta + \underbrace{4\sin^2\theta}_{2 - 2\cos 2\theta}) \, d\theta$$

$$= [9\theta + 12\cos\theta + 2\theta - \sin 2\theta]_{0}^{\pi}$$

$$= (9\pi - 12 + 2\pi) - (12)$$

$$= 11\pi - 24$$

(29) $r = 1 + \sin\theta$

$x = r\cos\theta$

$x = (1+\sin\theta)\cos\theta$

$$\frac{dx}{d\theta} = (1+\sin\theta)(-\sin\theta) + \cos^2\theta$$

Set $\frac{dx}{d\theta} = 0$: $0 = -\sin\theta - \sin^2\theta + \cos^2\theta$

$\cos^2\theta = 1-\sin^2\theta$

$$0 = -2\sin^2\theta - \sin\theta + 1$$

$$0 = (-2\sin\theta + 1)(\sin\theta + 1)$$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\sin\theta = -1$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$y = r\sin\theta$$

$$y = (1+\sin\theta)\sin\theta$$

$$y = \sin\theta + \sin^2\theta$$

$$\frac{dy}{d\theta} = \cos\theta + 2\sin\theta\cos\theta$$

Set $\frac{dy}{d\theta} = 0$: $0 = \cos\theta + 2\sin\theta\cos\theta$

$$0 = \cos\theta(1+2\sin\theta)$$

$\cos\theta = 0$ $\qquad$ $\sin\theta = -\frac{1}{2}$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Horizontal Tangent $\Rightarrow \frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} \neq 0$

$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Vertical Tangent $\Rightarrow \frac{dx}{d\theta} = 0$ and $\frac{dy}{d\theta} \neq 0$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$