Test 4

Mon Dec 4

9.9-9.10, 10.2-10.5, 12.1-12.2

6 Questions

Bring: calculator
music earplugs

Practice Problems on website

12.2 Derivatives and Integrals of Vector-Valued Functions Cont'd

Ex: $\vec{r}(t) = [\, t^2 \,,\, 7t \,,\, t^3 \,]$

Find $\vec{r}\,'(t) \cdot \vec{r}\,''(t)$

and $\vec{r}\,'(t) \times \vec{r}\,''(t)$

$\vec{r}\,'(t) = [\, 2t \,,\, 7 \,,\, 3t^2 \,]$

$\vec{r}\,''(t) = [\, 2 \,,\, 0 \,,\, 6t \,]$

$\vec{r}\,'(t) \cdot \vec{r}\,''(t) = 2t(2) + 7(0) + 3t^2(6t)$

$= 4t + 18t^3$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2t & 7 & 3t^2 \\ 2 & 0 & 6t \end{vmatrix}$$

$$= \vec{i}\,(42t) - \vec{j}\,(12t^2 - 6t^2) + \vec{k}\,(-14)$$

$$= 42t\,\vec{i} - 6t^2\,\vec{j} - 14\,\vec{k}$$

$$= [42t, -6t^2, -14]$$

FACT

If $\vec{r}(t) = [x(t), y(t), z(t)]$

then $\int \vec{r}(t)\,dt = \left[\int x(t)\,dt, \int y(t)\,dt, \int z(t)\,dt\right]$

Ex: Find $\vec{r}(t)$ if

$\vec{r}\,'(t) = [3t^2, 2t, 6e^{2t}]$

and $\vec{r}(0) = [3, 2, 8]$.

$$\vec{r}(t) = [t^3 + C_1, t^2 + C_2, 3e^{2t} + C_3]$$

$$= [t^3, t^2, 3e^{2t}] + \vec{C}$$

Sub $t=0$: $[3,2,8] = [0,0,3] + \vec{C}$

$\vec{C} = [3,2,5]$

$$\vec{r}(t) = [t^3, t^2, 3e^{2t}] + [3,2,5]$$

$$= [t^3+3, t^2+2, 3e^{2t}+5]$$

Ex: Find $\int_1^2 [4t, 7]\,dt$

$$= [2t^2, 7t]_1^2$$

$= [8, 14] - [2, 7]$

$= [6, 7]$

12.3 Velocity and Acceleration

Consider an object moving in 2D or 3D.

$\vec{r}(t)$ is the position vector

$\vec{v}(t)$ " velocity "

$\vec{a}(t)$ " acceleration "

$\|\vec{v}(t)\|$ is the speed (not a vector)

FACTS

$\vec{v}(t) = \vec{r}\,'(t)$

$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$

$\vec{v}(t) = \int \vec{a}(t)\,dt$

$\vec{r}(t) = \int \vec{v}(t)\,dt$

Ex: Given $\vec{r} = [t^3, t^2, t]$
Find the speed at $t = 1$.
Units are m and s.

$\vec{v} = [3t^2, 2t, 1]$

$\vec{v}(1) = [3, 2, 1]$

$\|\vec{v}(1)\| = \sqrt{3^2 + 2^2 + 1^2}$

$= \sqrt{14}$ m/s

Ex: Given $\vec{a} = [2, 4\cos 2t, 9e^{3t}]$,

$\vec{v}(0) = [1, -1, 0]$,

$\vec{r}(0) = [2, 1, 3]$.

Find $\vec{r}(t)$.

$$\vec{v} = [2t, 2\sin 2t, 3e^{3t}] + \vec{C}_1$$

Sub $t=0$: $[1,-1,0] = [0, 0, 3] + \vec{C}_1$

$$[1, -1, -3] = \vec{C}_1$$

$$\vec{v} = [2t+1, 2\sin 2t - 1, 3e^{3t} - 3]$$

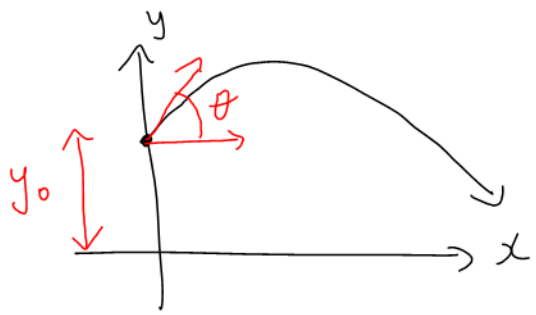
$$\vec{r} = [t^2 + t, -\cos 2t - t, e^{3t} - 3t] + \vec{C}_2$$

Sub $t=0$: $[2,1,3] = [0, -1, 1] + \vec{C}_2$

$$\vec{C}_2 = [2, 2, 2]$$

$$\vec{r} = [t^2 + t + 2, -\cos 2t - t + 2, e^{3t} - 3t + 2]$$

Projectile Motion in 2D



let v_0 = initial speed

θ = angle of inclination (measured to horizontal)

y_0 = initial height

g = 9.8 m/s^2 or 32 ft/s^2

Position (in m) after t seconds:

$$\vec{r} = \left[(v_0 \cos\theta)t \;,\; y_0 + (v_0 \sin\theta)t - \frac{gt^2}{2}\right]$$