Test Average : 80%

(2)

$$0 < \frac{0.3 + 0.6|\sin n|}{n^{1.2}} \leq \frac{1}{n^{1.2}} \quad \text{for } n \geq 1 \checkmark$$

and $\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}$ converges (p-series)

$$\Rightarrow \sum_{n=1}^{\infty} \frac{0.3 + 0.6|\sin n|}{n^{1.2}} \text{ converges}$$

(6)

$$\int_N^{\infty} \frac{1}{x^{2.5}}\,dx$$

$$= \lim_{b\to\infty} \int_N^b x^{-2.5}\,dx$$

$$= \lim_{b\to\infty} \frac{1}{-1.5} x^{-1.5} \Big|_N^b$$

$$= \lim_{b\to\infty} \frac{1}{-1.5} b^{-1.5} + \frac{1}{1.5} N^{-1.5}$$

$$= \frac{1}{1.5} N^{-1.5}$$

$$\frac{1}{1.5} N^{-1.5} \leq 0.1$$

$$\frac{1}{0.1(1.5)} \leq N^{1.5}$$

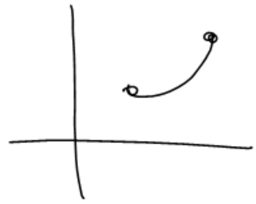
$$\left[\frac{1}{0.15}\right]^{\frac{1}{1.5}} \leq N$$

$$N \geq 3.5$$

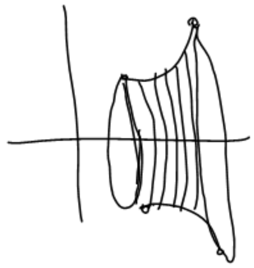
$$\boxed{N = 4}$$

10.3 Parametric Curves and Calculus

Consider $\begin{cases} x = x(t) \\ y = y(t) \\ a \le t \le b \end{cases}$

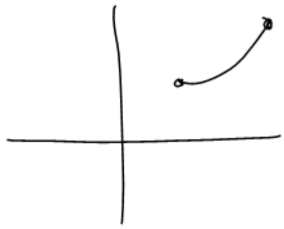


Revolve curve about x-axis:

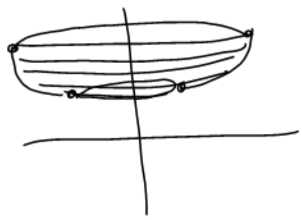


Surface

Surface Area $\quad S_x = 2\pi \int_a^b y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$



Revolve about y-axis:



Surface

Surface Area $\quad S_y = 2\pi \int_a^b x(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Ex: Revolve the curve about the x-axis.
Find the surface area.

$\begin{cases} x = 1 + t \\ y = 2\sqrt{t} \\ 0 \le t \le 4 \end{cases}$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = t^{-1/2}$$

$$S_x = 2\pi \int_a^b y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\pi \int_0^4 2\sqrt{t} \sqrt{1 + t^{-1}}\, dt$$

$$= 4\pi \int_0^4 \sqrt{t+1}\, dt$$

$$\boxed{\begin{array}{l} u = t+1 \\ du = dt \\ t=0 \Rightarrow u=1 \\ t=4 \Rightarrow u=5 \end{array}}$$

$$= 4\pi \int_1^5 \sqrt{u}\, du$$

$$= 4\pi \left[\frac{2}{3} u^{3/2}\right]_1^5$$

$$= \frac{8\pi}{3} \left[u^{3/2}\right]_1^5$$

$$= \frac{8\pi}{3} \left[5^{3/2} - 1\right]$$

Ex: Revolve the curve about the y-axis.
Find the surface area.

$$\cancel{\begin{cases} x = a\sin^3\theta \\ y = a\cos^3\theta \\ 0 \le \theta \le \frac{\pi}{2} \end{cases}}$$

$$\begin{cases} x = a\sin^3 t \\ y = a\cos^3 t \\ 0 \le t \le \frac{\pi}{2} \end{cases}$$

$$\frac{dx}{dt} = 3a\sin^2 t\cos t \qquad \Bigg| \qquad \frac{dy}{dt} = 3a\cos^2 t(-\sin t)$$

$$= -3a\cos^2 t\sin t$$

$$S_y = 2\pi\int_a^b x(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= 2\pi\int_0^{\pi/2} a\sin^3 t\sqrt{9a^2\sin^4 t\cos^2 t + 9a^2\cos^4 t\sin^2 t}\,dt$$

$$= 2\pi\int_0^{\pi/2} a\sin^3 t\sqrt{9a^2\sin^2 t\cos^2 t\,\cancel{(\sin^2 t + \cos^2 t)}}\,dt$$

$$= 2\pi\int_0^{\pi/2} a\sin^3 t\,|3a\sin t\cos t|\,dt$$

$$= 2\pi\int_0^{\pi/2} a\sin^3 t\,(3a\sin t\cos t)\,dt$$

$$= 6a^2\pi\int_0^{\pi/2}\sin^4 t\cos t\,dt$$

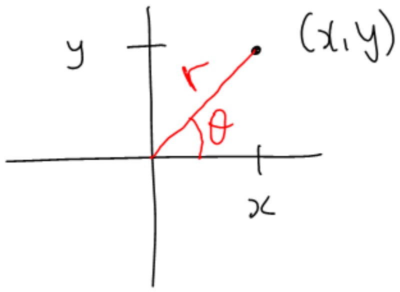
$$\boxed{\begin{array}{l} u = \sin t \\ du = \cos t\,dt \\ t = 0 \Rightarrow u = 0 \\ t = \frac{\pi}{2} \Rightarrow u = 1 \end{array}}$$

$$= 6a^2\pi\int_0^1 u^4\,du$$

$$= 6a^2\pi\left(\frac{u^5}{5}\right)\Bigg|_0^1$$

$$= \frac{6a^2\pi}{5}$$

10.4 Polar Coordinates and Graphs



Rectangular Cordinates (x, y)
Polar Cordinates (r, θ)

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\frac{y}{x}$ $(+\pi ?)$

Add π when $x < 0$

Ex: Convert $(r, \theta) = (6, \frac{\pi}{6})$ to rectangular.

$x = r\cos\theta = 6\cos\frac{\pi}{6} = 6\left(\frac{\sqrt{3}}{2}\right) = 3\sqrt{3}$

$y = r\sin\theta = 6\sin\frac{\pi}{6} = 3$

$(x, y) = (3\sqrt{3}, 3)$

Ex: Convert $(x, y) = (-\sqrt{3}, 1)$ to polar.

$r = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$

$\theta = \tan^{-1}\frac{y}{x}$ $(+\pi ?)$

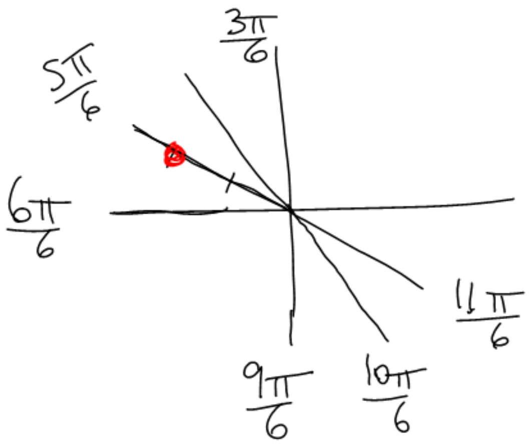
$$= \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) + \pi$$

$$= -\frac{\pi}{6} + \pi$$

$$= \frac{5\pi}{6}$$

$$(r, \theta) = \left(2, \frac{5\pi}{6}\right)$$

Ex: Describe $(r, \theta) = \left(2, \frac{5\pi}{6}\right)$ with a negative r-value.

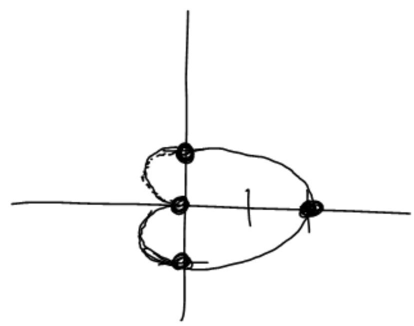


$$(r, \theta) = \left(-2, \frac{11\pi}{6}\right) \checkmark$$

$$\text{or } \left(-2, -\frac{\pi}{6}\right) \checkmark$$

Ex: Sketch $r = 1 + \cos\theta$

θ	$r = 1 + \cos\theta$
0	2
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	1
2π	2

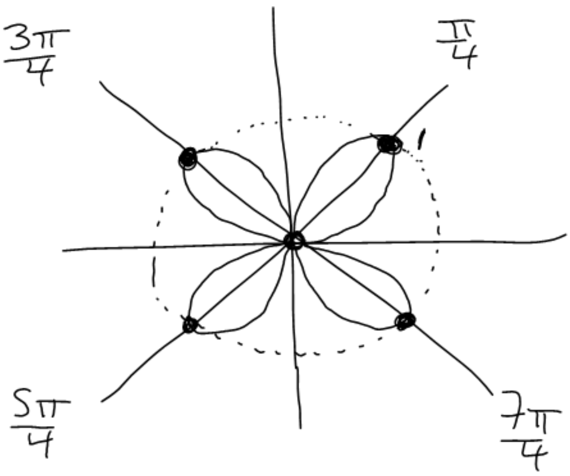


limaçon
(Pronounced LI-MA-SO)

Ex: Sketch $r = \sin 2\theta$

θ	$r = \sin 2\theta$
0	0
$\frac{\pi}{4}$	1
$\frac{2\pi}{4}$	0
$\frac{3\pi}{4}$	-1
$\frac{4\pi}{4}$	0

θ	$r = \sin 2\theta$
$\frac{5\pi}{4}$	1
$\frac{6\pi}{4}$	0
$\frac{7\pi}{4}$	-1
$\frac{8\pi}{4}$	0



rose with 4 petals