Test 3

FRI NOV 10
9.2-9.8
6 Questions
Bring: calculator, music/earplugs
Practice Problems on website

Ch 10
Parametric Curves
Polar Curves

10.2 Parametric Curves

Parametric curve

$$\begin{cases} x = f(t) \\ y = g(t) \\ a \le t \le b \end{cases}$$

t is the parameter

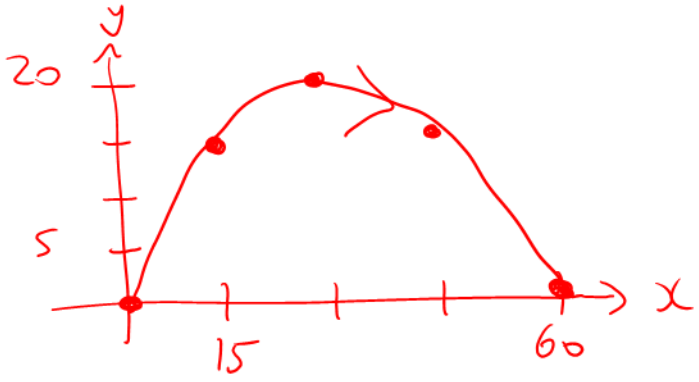
A parametric curve is a set of equations that trace out a curve as t varies.

Ex: Plot $\begin{cases} x = 15t \\ y = 20t - 5t^2 \\ 0 \le t \le 4 \end{cases}$

t	x	y
0	0	0

1	15	15
2	30	20
3	45	15
4	60	0

Plot



Ex: Eliminate the parameter

$$\begin{cases} x = 15t \\ y = 20t - 5t^2 \end{cases} \quad (-\infty < t < \infty)$$

Solve for t: $x = 15t$

$$\frac{x}{15} = t$$

$t = \frac{x}{15} \rightarrow y = 20t - 5t^2$

$$y = 20\left(\frac{x}{15}\right) - 5\left(\frac{x}{15}\right)^2 \quad \checkmark$$

$$y = \frac{4x}{3} - \frac{5x^2}{225} \quad \frac{x^2}{45} \quad \checkmark$$

Ex: Eliminate the parameter

$$\begin{cases} x = h + r\cos t \\ y = k + r\sin t \\ 0 \le t < 2\pi \end{cases}$$

$(h, k, r$: constants$)$

$x - h = r\cos t$ | $y - k = r\sin t$

$\frac{x-h}{r} = \cos t$ | $\frac{y-k}{r} = \sin t$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x-h}{r}\right)^2 + \left(\frac{y-k}{r}\right)^2 = 1 \quad \checkmark$$

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1 \quad \checkmark$$

$$(x-h)^2 + (y-k)^2 = r^2 \quad \checkmark$$

CIRCLE

Ex: Eliminate the parameter

$$\begin{cases} x = h + a\cos t \\ y = k + b\sin t \\ 0 \le t < 2\pi \end{cases}$$

$(h, k, a, b$: Constants)

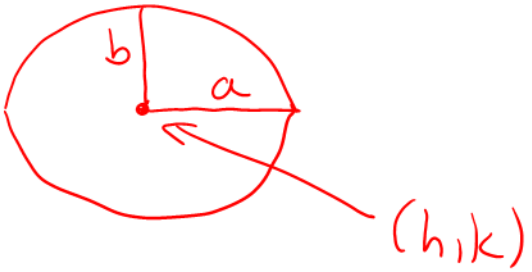
$x - h = a\cos t$ | $y - k = b\sin t$

$\frac{x-h}{a} = \cos t$ | $\frac{y-k}{b} = \sin t$

$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 1$$ ✓

ELLIPSE



EX: Write as a parametric curve:

$$(x-2)^2 + (y+5)^2 = 9$$

$$\frac{(x-2)^2}{9} + \frac{(y+5)^2}{9} = 1$$

$$\left(\frac{x-2}{3}\right)^2 + \left(\frac{y+5}{3}\right)^2 = 1$$

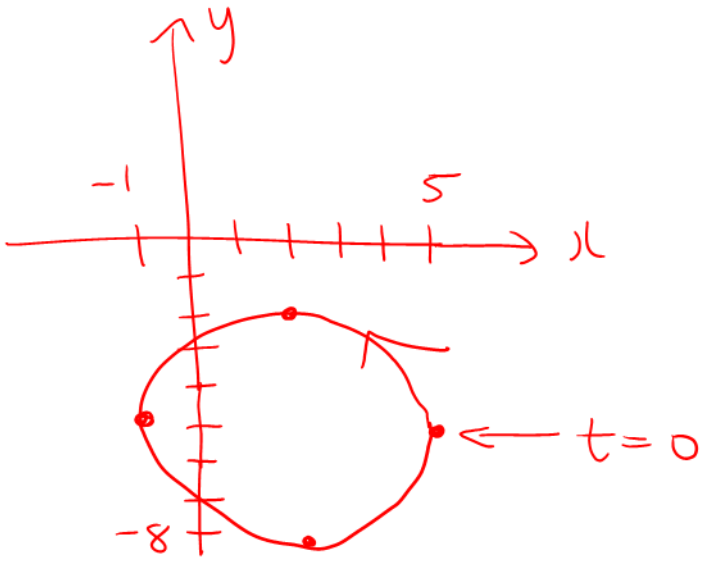
$\cos t = \frac{x-2}{3}$ | $\sin t = \frac{y+5}{3}$

$3\cos t = x - 2$ | $3\sin t = y + 5$

$x = 3\cos t + 2$ | $y = 3\sin t - 5$

$$\begin{cases} x = 2 + 3\cos t \\ y = -5 + 3\sin t \end{cases} \qquad 0 \le t < 2\pi$$

t	x	y
0	5	-5
$\pi/2$	2	-2
π	-1	-5
$3\pi/2$	2	-8



Parametrization of a curve is not unique.

Ex: Give 3 parametrizations of the line segment from $(2,5)$ to $(4,8)$.

a) direction vector $\vec{d} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} t$$

$$\begin{cases} x = 2 + 2t \\ y = 5 + 3t \end{cases} \qquad 0 \le t \le 1$$

b) Speed it up.

$$\begin{cases} 0 \le t \le 0.5 \\ x = 2 + \underline{\underline{4t}} \\ y = 5 + \underline{\underline{6t}} \end{cases}$$

c) Slow it down.

$$\begin{cases} 0 \le t \le 2 \\ x = 2 + \underline{\underline{t}} \\ y = 5 + 1.5\underline{\underline{t}} \end{cases}$$