Suggested HW pdf is on D2L
Answers at back of pdf file
List of problems on website

Test 2 ~~Fri Oct 20~~ Mon Oct 23
Test 4 ~~Fri Dec 1~~ Mon Dec 4

Section 1.3 #51

$$\lim_{x \to -3} \frac{x^2+x-6}{x^2-9}$$

$$= \lim_{x \to -3} \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-3)}$$

$$= \frac{-5}{-6}$$

$$= \frac{5}{6}$$

Section 1.5 #51

$$\lim_{x \to 1^+} \frac{x^2+x+1}{x^3-1}$$

$$= \frac{3}{0^+}$$

$$= +\infty$$

Section 2.3 #49

Find y' for $y = \dfrac{3(1-\sin x)}{2\cos x}$

$$y' = \frac{(2\cos x)[3(-\cos x)] - 3(1-\sin x)(-2\sin x)}{4\cos^2 x}$$

$$= \frac{-6\cos^2 x + 6\sin x(1-\sin x)}{4\cos^2 x}$$

$$= \frac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{4\cos^2 x}$$ ✓

$$= \frac{-6 + 6\sin x}{4\cos^2 x}$$ ✓

Section 2.4 #19

Find $g'(s)$ if $g(s) = \dfrac{6}{(s^3-2)^3}$

$$g(s) = 6(s^3-2)^{-3}$$

$$g'(s) = -18(s^3-2)^{-4}(3s^2)$$

$$= \frac{-54s^2}{(s^3-2)^4}$$

2.2-2.4 Derivative Rules and Trig Cont'd

Ex: Find the second derivative of $y = \sin 3x^2$

$$y' = \cos 3x^2 \,(6x)$$

$$= 6x \cos 3x^2$$

$$y'' = 6x\left[-\sin 3x^2 \,(6x)\right] + \left(\cos 3x^2\right)(6)$$

$$= -36x^2 \sin 3x^2 + 6\cos 3x^2$$

Ex: Find an equation for the tangent line to $y = 2x^3 + 5x^2 - 1$ at $x = 1$

$$y' = 6x^2 + 10x$$

$$y'\big|_{x=1} = 16$$

$m = 16 \qquad x_1 = 1 \qquad x = 1 \rightarrow y = 6$

$y_1 = 6$

$$y - y_1 = m(x - x_1)$$

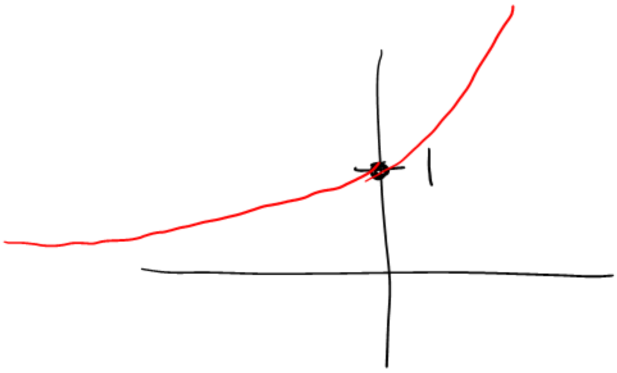
$y - 6 = 16(x-1)$ ✓

5.1 Derivatives of Exponentials and Logs

Exponential Function:

$f(x) = b^x$ b: constant

The most important base is $e \simeq 2.718$



$y = e^x$

$$\frac{d}{dx} e^x = e^x$$

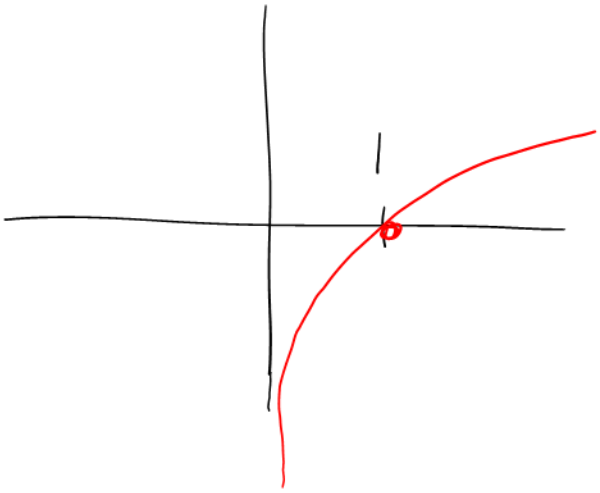
$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\boxed{e^a e^b = e^{a+b} \qquad (e^a)^b = e^{ab} \qquad \frac{e^a}{e^b} = e^{a-b}}$$

Logarithmic Function:

$f(x) = \log_b x \qquad b:$ constant

Note: $\log_e x$ is written $\ln x$



$y = \ln x$

<u>Ex</u>: Find $\lim_{x \to 7^+} \ln(x-7)$

$= \ln 0^+$

$= -\infty$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$$\boxed{\begin{aligned} \ln(ab) &= \ln a + \ln b \\ \ln\left(\frac{a}{b}\right) &= \ln a - \ln b \\ \ln a^b &= b \ln a \end{aligned}}$$

Ex: Find y'

a) $y = e^{3x}$

$$y' = e^{3x}(3)$$
$$= 3e^{3x}$$

b) $y = \ln(x^3+1)$

$$y' = \frac{1}{x^3+1}(3x^2)$$
$$= \frac{3x^2}{x^3+1}$$

c) $y = e^{-2x} \sin 5x$

$$y' = e^{-2x}[\cos 5x\,(5)] + (\sin 5x)[e^{-2x}(-2)]$$
$$= 5e^{-2x}\cos 5x - 2e^{-2x}\sin 5x \checkmark$$

$$= e^{-2x}(5\cos 5x - 2\sin 5x)$$

d) $y = \ln\sqrt{\dfrac{x+1}{2x+3}}$

$$y = \frac{1}{2}\ln\left(\frac{x+1}{2x+3}\right)$$

$$y = \frac{1}{2}\left[\ln(x+1) - \ln(2x+3)\right]$$

$$y' = \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{2x+3}(2)\right]$$

$$= \frac{1}{2}\left[\frac{1}{x+1} - \frac{2}{2x+3}\right]$$

e) $y = e^{x^2} + \cos(\ln x)$

$$y' = e^{x^2}(2x) - \sin(\ln x)\frac{1}{x}$$

$$= 2xe^{x^2} - \frac{\sin(\ln x)}{x}$$

<u>Ex</u>: Let $y = e^{2x} - 5x$

Find x so that $y' = 0$.

$$y' = e^{2x}(2) - 5$$

$$y' = 2e^{2x} - 5$$

Set $y' = 0$: $2e^{2x} - 5 = 0$

$$2e^{2x} = 5$$

$$e^{2x} = \frac{5}{2}$$

Take ln: $2x = \ln\left(\frac{5}{2}\right)$

$$x = \frac{1}{2}\ln\left(\frac{5}{2}\right)$$

2.5 Implicit Differentiation

y is an <u>explicit</u> function of x:

$$y = \pm\sqrt{25 - x^2}$$

y is an <u>implicit</u> function of x:

$$x^2 + y^2 = 25$$

<u>Ex</u>: y depends on x

Find:

a) $\frac{d}{dx}[x^2] = 2x$

b) $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$ Chain Rule

c) $\frac{d}{dx}[(4x^2)y^3]$

$$= 4x^2\left(3y^2\frac{dy}{dx}\right) + y^3(8x)$$

$$= 12x^2y^2\frac{dy}{dx} + 8xy^3$$

Ex: Find $\frac{dy}{dx}$ given $x^2+y^2=25$

1) Take $\frac{d}{dx}$:

$$2x + 2y\frac{dy}{dx} = 0$$

2) Solve for $\frac{dy}{dx}$:

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$= \frac{-x}{y}$$