2.2-2.4 Derivative Rules and Trig Cont'd

Power Rule $\frac{d}{dx}[x^n] = nx^{n-1}$

Product Rule $[fg]' = fg' + gf'$

Quotient Rule $\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$

Chain Rule: Calculation Version

$$[f(g(x))]' = f'(g(x))\,g'(x)$$

Chain Rule: Formal Version

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Ex: $y = \sqrt[3]{x^3+1}$. Find y'.

$$y = (x^3+1)^{1/3}$$

$$y' = \frac{1}{3}(x^3+1)^{-2/3}(3x^2)$$

$$= \frac{x^2}{\sqrt[3]{(x^3+1)^2}}$$

Ex: Confirm using the formal Chain Rule.

$$y = \sqrt[3]{u} \qquad u = x^3 + 1$$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

$$= \frac{1}{3}u^{-2/3}\,(3x^2)$$

$$= \frac{1}{3}(x^3+1)^{-2/3}\,(3x^2) \checkmark$$

Ex: Find $\frac{dy}{dx}$

a) $y = 5x^4 + \frac{3}{x^2} + 6\sqrt{x}$

$$y = 5x^4 + 3x^{-2} + 6x^{1/2}$$

$$\frac{dy}{dx} = 20x^3 - 6x^{-3} + 3x^{-1/2}$$

$$\text{or } 20x^3 - \frac{6}{x^3} + \frac{3}{\sqrt{x}}$$

b) $y = \frac{x^3}{2x+1}$

$$\frac{dy}{dx} = \frac{(2x+1)(3x^2) - x^3(2)}{(2x+1)^2}$$

$$= \frac{4x^3 + 3x^2}{(2x+1)^2}$$

Ex: Find $f'(1)$ for

$$f(x) = x^2(x^2+5x+1)(x^7+x^3+6)$$

$$f(x) = (x^4+5x^3+x^2)(x^7+x^3+6)$$

$$f'(x) = (x^4+5x^3+x^2)(7x^6+3x^2) + (x^7+x^3+6)(4x^3+15x^2+2x)$$

$$f'(1) = (7)(10) + (8)(21)$$

$$= 238$$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$

Ex: Find y'

a) $y = \csc x^2$

$$y' = -\csc x^2 \cot x^2 \;(2x)$$
$$= -2x \csc x^2 \cot x^2$$

b) $y = \csc^2 x$

$$y = [\csc x]^2$$
$$y' = 2[\csc x](-\csc x \cot x)$$
$$= -2\csc^2 x \cot x$$

Ex: Why does $\frac{d}{dx}[\tan x] = \sec^2 x$?

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$
$$= \frac{(\cos x)(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$
$$= \frac{1}{\cos^2 x}$$
$$= \left[\frac{1}{\cos x}\right]^2$$
$$= \sec^2 x$$

Ex: Find $\dfrac{df}{dx}$

a) $f = \sin 5x$

$$\frac{df}{dx} = \cos 5x \,(5)$$
$$= 5\cos 5x$$

b) $f = x \tan x^2$

$$\frac{df}{dx} = x\left[\sec^2 x^2 \,(2x)\right] + \tan x^2 \,(1)$$
$$= 2x^2 \sec^2 x^2 + \tan x^2$$

c) $f = \sin^3 x + \cos^3 x \leftarrow [\cos x]^3$

$$f = [\sin x]^3 + [\cos x]^3$$
$$\frac{df}{dx} = 3[\sin x]^2(\cos x) + 3[\cos x]^2(-\sin x)$$
$$= 3\sin^2 x \cos x - 3\cos^2 x \sin x$$

d) $f = \sec^2 x^3$

$$f = [\sec x^3]^2$$
$$\frac{df}{dx} = 2[\sec x^3]\left[\sec x^3 \tan x^3 \,(3x^2)\right]$$
$$= 6x^2 \sec^2 x^3 \tan x^3$$