

www.leahhoward.com

→ Math 250B

Miss a test \Rightarrow weight shifts to exam

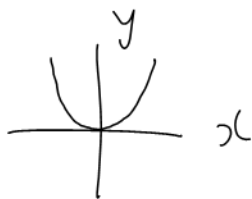
List of Sugg HW Problems
and Full Solutions on website

Problems are on D2L

Overview of Multivariable Calculus

single-variable function

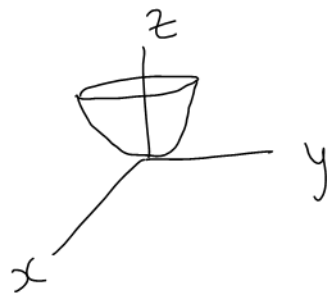
$$y = x^2$$



curve

multi-variable function

$$z = x^2 + y^2$$

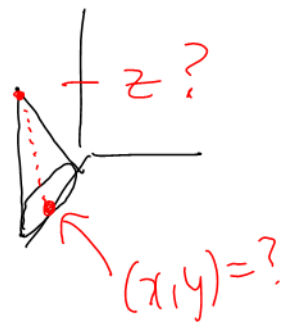


surface

(Imagine an infinitely-thin
piece of fabric.)

I Differentiation

Maximize/minimize a function

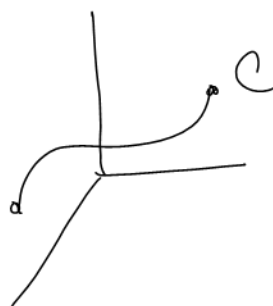
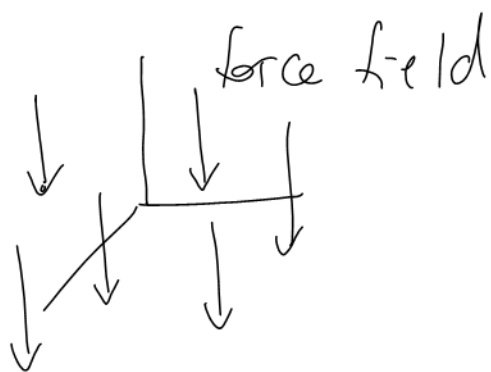


II Integration

Volume under a surface



III Vector Fields



Work done by the force field on a particle travelling along c ?

11.7/12.1/12.2 Surfaces and Functions of Several Variables

Function of several variables:

Input is 2 or more variables

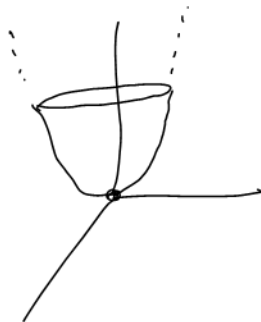
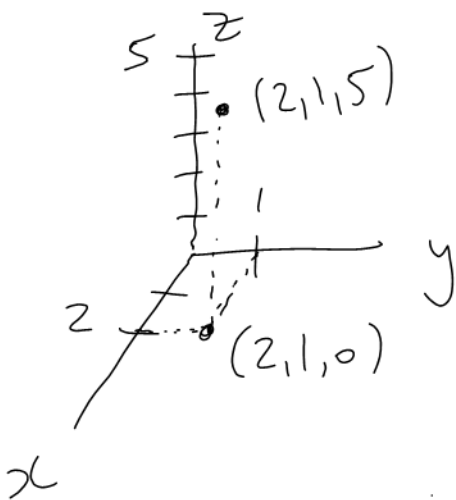
e.g. $f(x,y) = x^2 + y^2$

$$f = x^2 + y^2$$

$$z = x^2 + y^2$$

Evaluating: $f(2,1) = 5$

As a point $(x,y,z) = (2,1,5)$

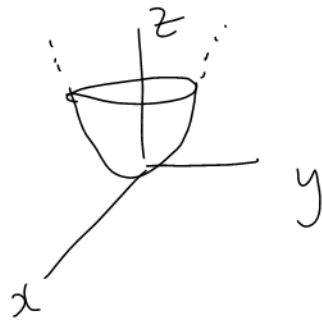


$$z = x^2 + y^2$$

2D surface
in 3D space

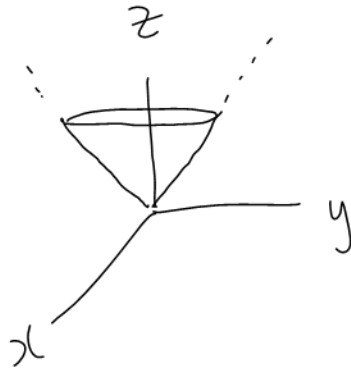
Five Important Surfaces

$$z = x^2 + y^2$$



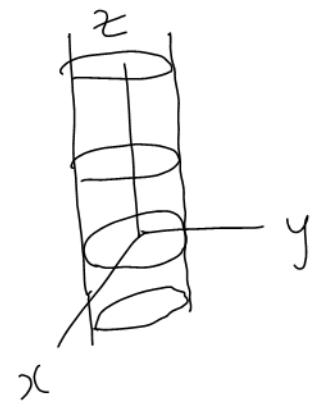
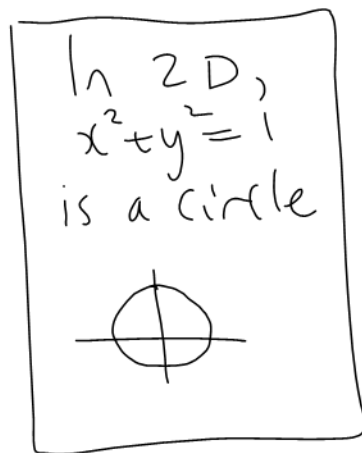
paraboloid

$$z = \sqrt{x^2 + y^2}$$



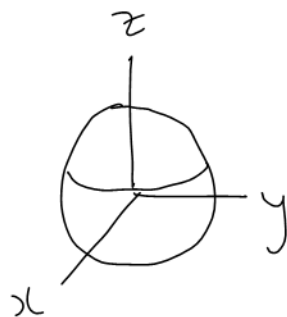
Cone

$$x^2 + y^2 = 1$$



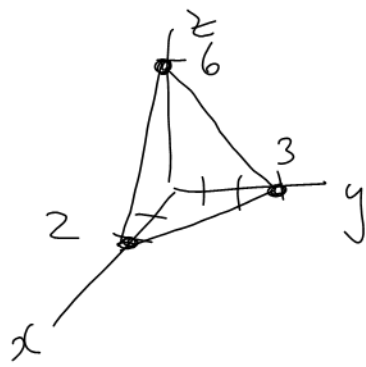
$x^2 + y^2 = 1$
(circular)
cylinder

$$x^2 + y^2 + z^2 = 1$$



Sphere

$$3x + 2y + z = 6$$



plane

A plane is an infinite surface.

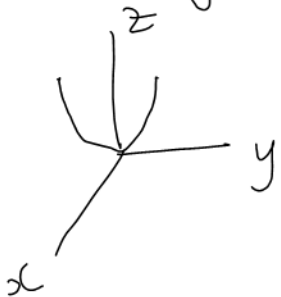
Graphing Paraboloids

Trace : Surface \cap Plane,
especially when the plane is
 $x=0$, $y=0$, or $z=0$.

Ex : Graph $z = x^2 + y^2$ using traces.

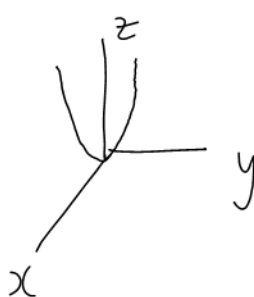
$$x=0:$$

$$z = y^2$$



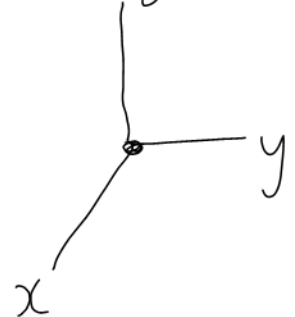
$$y=0:$$

$$z = x^2$$



$$z=0:$$

$$0 = x^2 + y^2$$



$$z = x^2 + y^2$$

paraboloid

Contour curve : Surface n ($z=c$)

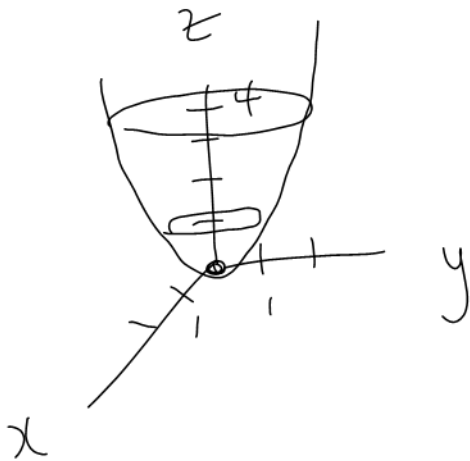
Ex: Graph $z = x^2 + y^2$ using contour curves.

$z=0$: $0 = x^2 + y^2$ circle of radius 0

$z=1$: $1 = x^2 + y^2$ " 1

$z=4$: $4 = x^2 + y^2$ " 2

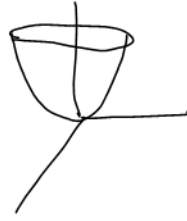
$z=-1$: $-1 = x^2 + y^2$ no solution
(does not exist)



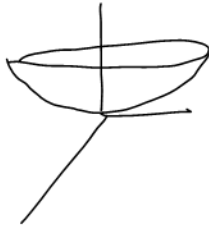
paraboloid

Ex: Transformations of $z = x^2 + y^2$

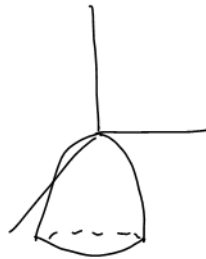
$$z = x^2 + y^2$$



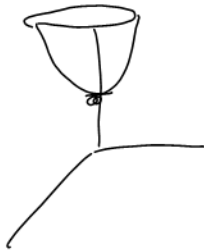
$$z = \frac{1}{2}(x^2 + y^2)$$



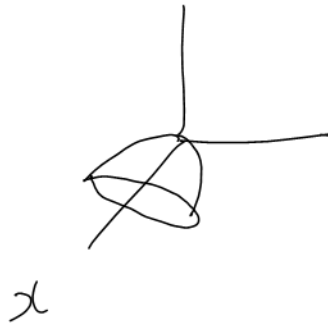
$$z = -(x^2 + y^2)$$



$$z = x^2 + y^2 + 1$$



$$x = y^2 + z^2$$



Graphing Spheres

Ex: Graph $x^2 + y^2 + z^2 = 9$ using contour curves.

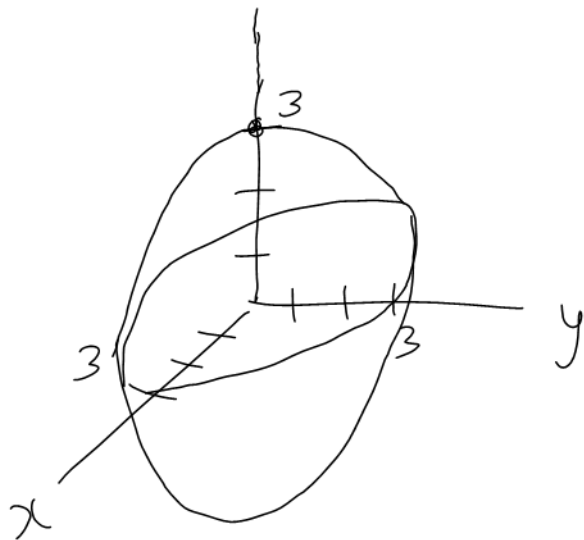
$z = 0$: $x^2 + y^2 = 9$ circle of radius 3

$z = \pm 1$: $x^2 + y^2 = 8$ " $\sqrt{8}$

$z = \pm 2$: $x^2 + y^2 = 5$ " $\sqrt{5}$

$z = \pm 3$: $x^2 + y^2 = 0$ " 0

$z = \pm 4$: $x^2 + y^2 + 16 = 9$ no solution



sphere