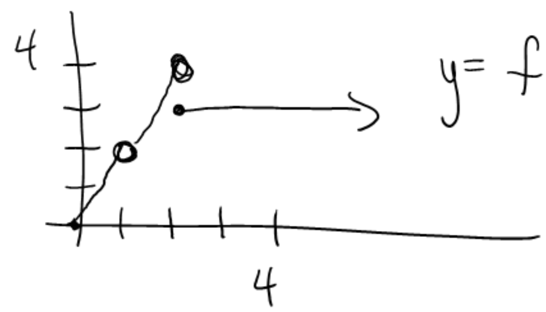
Math 250A Website
www.leahhoward.com

Suggested HW pdf is on D2L.
Answers at the back.
List of problems is on website.
Full solutions linked on website.

1.2-1.5 Limits and Continuity



$$\lim_{x\to 1} f(x) = 2$$

Means: As x approaches 1, $f(x)$ approaches 2.
Behaviour at $x=1$ is irrelevant.

$$\lim_{x\to 4} f(x) = 3$$

$\lim_{x\to 2} f(x)$ does not exist

Limit from the left $\lim_{x\to 2^-} f(x) = 4$
" right $\lim_{x\to 2^+} f(x) = 3$

"one-sided limits"

Def

$f(x)$ is continuous at $x=a$ if

$$\lim_{x\to a} f(x) = f(a)$$

[no hole/jump in graph at $x=a$]

Def

$f(x)$ is continuous on an interval if $f(x)$ is continuous at each x-value in the interval.

Ex: Find $\displaystyle\lim_{x\to 2} \frac{2x+1}{x+1}$

$$= \frac{5}{3}$$

Can plug in when the function is continuous at the x-value.

Ex: Find $\displaystyle\lim_{x\to 2} \frac{x^2-4}{x^2-x-2}$

$\boxed{\frac{0}{0} \text{ gives no info}}$

$$= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(x+1)}$$

$$= \lim_{x\to 2} \frac{x+2}{x+1}$$

$$= \frac{4}{3}$$

Recall $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Ex: Find $\lim_{x \to 2} \frac{x^3 - 8}{x^4 - 16}$

$$= \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{(x^2-4)(x^2+4)}$$

$$= \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)(x^2+4)}$$

$$= \lim_{x \to 2} \frac{x^2+2x+4}{(x+2)(x^2+4)}$$

$$= \frac{12}{32} \text{ or } \frac{3}{8}$$

Ex: Find $\lim_{x \to 6} \frac{\sqrt{x+3} - 3}{x-6}$

$$= \lim_{x \to 6} \frac{(\sqrt{x+3} - 3)}{(x-6)} \frac{(\sqrt{x+3} + 3)}{(\sqrt{x+3} + 3)}$$

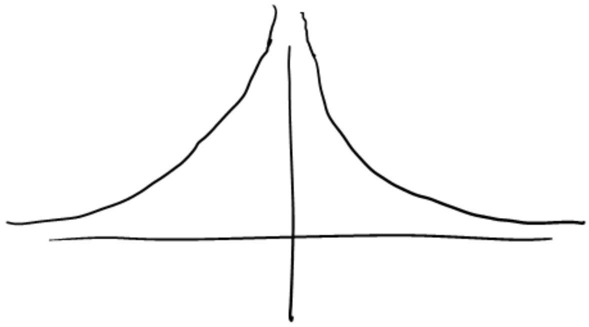
$$= \lim_{x \to 6} \frac{x+3 - 9}{(x-6)(\sqrt{x+3} + 3)}$$

$$= \lim_{x \to 6} \frac{\cancel{x-6} \; 1}{\cancel{(x-6)}(\sqrt{x+3} + 3)}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$

Ex: Find $\lim_{x\to 0} \frac{1}{x^2}$ and $\lim_{x\to -\infty} \frac{1}{x^2}$



$y = \frac{1}{x^2}$

$$\lim_{x\to 0} \frac{1}{x^2} = \infty$$ ⟵ more precise

OR $\lim_{x\to 0} \frac{1}{x^2}$ does not exist

$$\lim_{x\to -\infty} \frac{1}{x^2} = 0$$

Ex: Find $\lim_{x\to 3^+} \frac{2x}{9-x^2}$

$$= \frac{6}{0^-}$$

$$= -\infty$$

FACT

$\lim_{x\to \infty} \frac{1}{x^N} = 0$ and $\lim_{x\to -\infty} \frac{1}{x^N} = 0$

for any $N > 0$.

Ex: Find $\lim_{x\to\infty} \frac{3x^2+5x+1}{5x^2+7}$

$\boxed{\frac{\infty}{\infty} \text{ gives no info}}$

$$= \lim_{x\to\infty} \frac{x^2\left(3+\frac{5}{x}+\frac{1}{x^2}\right)}{x^2\left(5+\frac{7}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{3+\frac{5}{x}+\frac{1}{x^2}}{5+\frac{7}{x^2}}$$

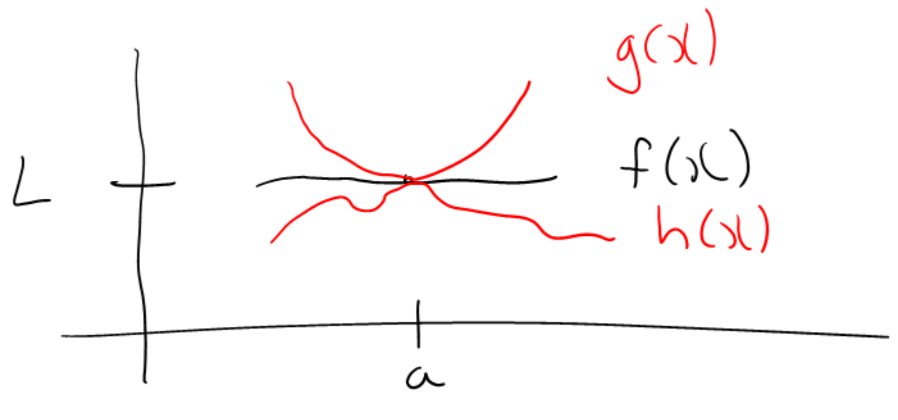
$$= \frac{3}{5}$$

Squeeze Theorem

Suppose $h(x) \le f(x) \le g(x)$
for all x near $x=a$.

If $\lim_{x\to a} h(x) = L$ and $\lim_{x\to a} g(x) = L$

then $\lim_{x\to a} f(x) = L$.



Ex: Find $\lim_{x \to 0} x^2 \sin \frac{1}{x}$.

$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$

$$\lim_{x \to 0} -x^2 = 0 \quad \text{and} \quad \lim_{x \to 0} x^2 = 0$$

Conclude $\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$

2.2-2.4 Derivative Rules and Trig

The derivative of $y = f(x)$ can be written:

$$y', \quad f'(x), \quad \frac{dy}{dx} \quad \text{or} \quad \frac{df}{dx}$$

Evaluating:

$$y'\Big|_{x=a}, \quad f'(a), \quad \frac{dy}{dx}\Big|_{x=a} \quad \text{or} \quad \frac{df}{dx}\Big|_{x=a}$$

$f'(x)$ represents:

slope of the tangent line

and instantaneous rate of change of $f(x)$