Test 1

5 Questions

Covers 1.1-1.4, 2.1-2.2

Tues Sept 23, 1:30-2:20

Bring calculator and music/earplugs

Practice Problems on website

No formula sheet

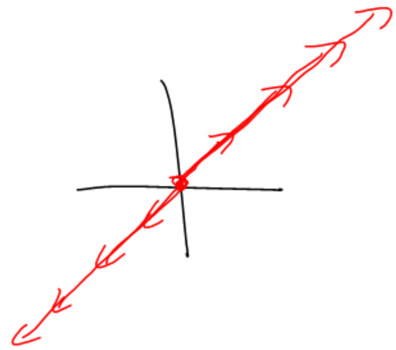
Example: Describe each span geometrically:

a) span($\begin{bmatrix}1\\1\end{bmatrix}$, $\begin{bmatrix}-3\\-3\end{bmatrix}$)

$= \left\{ c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}-3\\-3\end{bmatrix} \right\}$

$= \left\{ c_1\begin{bmatrix}1\\1\end{bmatrix} \right\}$

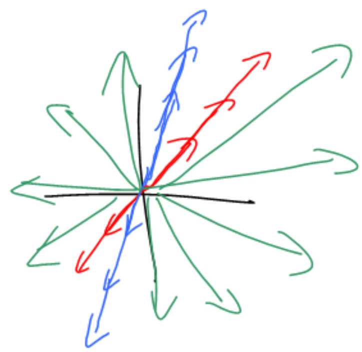
Line in $\mathbb{R}^2$ through the origin with $\vec{d} = \begin{bmatrix}1\\1\end{bmatrix}$.



b) span($\begin{bmatrix}1\\1\end{bmatrix}$, $\begin{bmatrix}1\\2\end{bmatrix}$)

$= \left\{ c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}1\\2\end{bmatrix} \right\}$

All of $\mathbb{R}^2$.

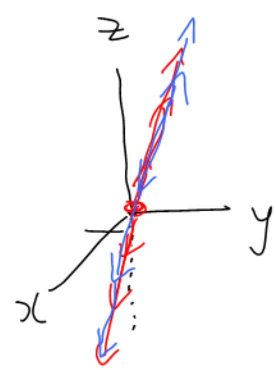


c) span($\begin{bmatrix}1\\0\\-1\end{bmatrix}$, $\begin{bmatrix}-4\\0\\4\end{bmatrix}$)

$= \left\{ c_1\begin{bmatrix}1\\0\\-1\end{bmatrix} + c_2\begin{bmatrix}-4\\0\\4\end{bmatrix} \right\}$

$= \left\{ c_1\begin{bmatrix}1\\0\\-1\end{bmatrix} \right\}$

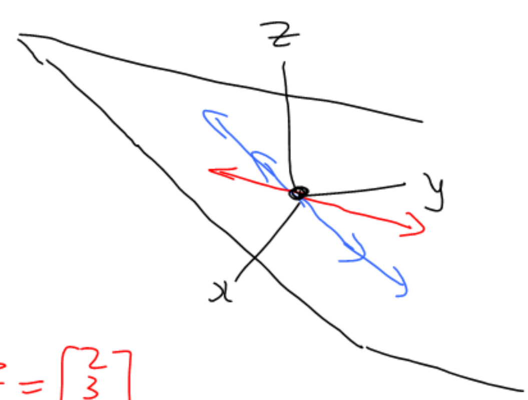
Line in $\mathbb{R}^3$ through the origin with $\vec{d} = \begin{bmatrix}1\\0\\-1\end{bmatrix}$.



d) span($\begin{bmatrix}1\\6\\0\end{bmatrix}$, $\begin{bmatrix}2\\3\\0\end{bmatrix}$)

$= \left\{ c_1\begin{bmatrix}1\\6\\0\end{bmatrix} + c_2\begin{bmatrix}2\\3\\0\end{bmatrix} \right\}$

Plane in $\mathbb{R}^3$ through the origin with direction vectors $\vec{u} = \begin{bmatrix}1\\6\\0\end{bmatrix}$, $\vec{v} = \begin{bmatrix}2\\3\\0\end{bmatrix}$



Example: Find an equation for span($\begin{bmatrix}1\\0\\1\end{bmatrix}$, $\begin{bmatrix}1\\3\\6\end{bmatrix}$). Give your answer in any form.

It's a plane (in $\mathbb{R}^3$) through the origin with direction vectors $\vec{u}=\begin{bmatrix}1\\0\\1\end{bmatrix}$ and $\vec{v}=\begin{bmatrix}1\\3\\6\end{bmatrix}$.

Normal Form

$$\vec{n}=\begin{bmatrix}1\\0\\1\end{bmatrix}\times\begin{bmatrix}1\\3\\6\end{bmatrix}=\begin{bmatrix}-3\\-5\\3\end{bmatrix}$$

$$\boxed{\begin{matrix}1 & 0 & 1 & 1 & 0\\ & \times & \times & \times & \\ 1 & 3 & 6 & 1 & 3\end{matrix}}$$

$$\vec{n}\cdot\vec{x}=\vec{n}\cdot\vec{p}$$

$$\begin{bmatrix}-3\\-5\\3\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-3\\-5\\3\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\end{bmatrix} \checkmark$$

OR Vector Form

$$\vec{x}=\vec{p}+s\vec{u}+t\vec{v}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}+s\begin{bmatrix}1\\0\\1\end{bmatrix}+t\begin{bmatrix}1\\3\\6\end{bmatrix} \checkmark$$

OR General Form

$$-3x-5y+3z=0 \checkmark$$

OR Parametric Form

$$\begin{cases}x= & s+t\\ y= & 3t\\ z= & s+6t\end{cases} \checkmark$$

Example: a) Show that span($\begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}$)$= \mathbb{R}^2$.

Let $c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$.

Show that this is a consistent system.

$$\begin{array}{cc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & a \\ 3 & 1 & b \end{array}\right]$$

$$R_2 - 3R_1 \quad \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & -5 & b-3a \end{array}\right] \quad \text{REF}$$

The system is solvable for c_1, c_2.

b) Write $\begin{bmatrix}a\\b\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\3\end{bmatrix}$ and $\begin{bmatrix}2\\1\end{bmatrix}$.

Let $c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$

$\vdots$

$$\left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & -5 & b-3a \end{array}\right]$$

$$\frac{R_2}{-5} \quad \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & 1 & \frac{3a}{5} - \frac{b}{5} \end{array}\right] \leftarrow \frac{b-3a}{-5} = \frac{b}{-5} + \frac{-3a}{-5}$$

$$R_1 - 2R_2 \quad \left[\begin{array}{cc|c} 1 & 0 & \frac{-a+2b}{5} \\ 0 & 1 & \frac{3a-b}{5} \end{array}\right] \begin{array}{l} \leftarrow a - 2\left(\frac{3a}{5} - \frac{b}{5}\right) \\ = a - \frac{6a}{5} + \frac{2b}{5} \\ = \frac{-a}{5} + \frac{2b}{5} \end{array}$$

RREF

$$c_1 = \frac{-a+2b}{5} \qquad c_2 = \frac{3a-b}{5}$$

$$\boxed{\left(\frac{-a+2b}{5}\right)\begin{bmatrix}1\\3\end{bmatrix} + \left(\frac{3a-b}{5}\right)\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}}$$

(Solvable)

Comment: To decide if a system is consistent, reduce it to REF.
To solve a system, reduce it to RREF.

Definition: Given $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$, consider solutions to $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$. If the only solution is $c_1 = c_2 = \ldots = c_n = 0$ then the set of vectors is **linearly independent**. If there are solutions other than $c_1 = c_2 = \ldots = c_n = 0$ then the set of vectors is **linearly dependent**.

$$c_1\vec{v}_1 + \ldots + c_n\vec{v}_n = \vec{0}$$

$c_1 = c_2 = \ldots = c_n = 0$ is the only solution

"vectors are linearly independent"

$c_1 = c_2 = \ldots = c_n = 0$ and other solutions

"vectors are linearly dependent"

Comment: The two sentences below mean the same thing:
Vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are linearly independent.
The set $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is linearly independent.

Comment: The two sentences below mean the same thing:
Vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are linearly dependent.
The set $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is linearly dependent.