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Skeleton Notes can be printed at the Printshop.

If you a miss a test then weight shifts to the exam.

Course Overview

Matrix Algebra is also known as “Linear Algebra” or “Algebra and Geometry.”

A geometry problem could involve visualizing lines and planes in 3D space.

An algebra problem could involve calculating distances and angles, especially in higher dimensions.

Many problems in Matrix Algebra involve the interplay of geometry and algebra.

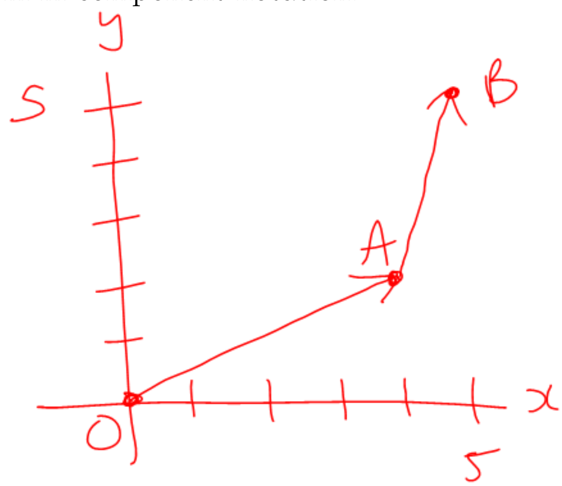
Why do we need higher dimensions? Tracking an object’s spatial location and temperature is a 4D problem.

Chapter 1: Vectors

1.1 The Geometry and Algebra of Vectors

Definition: A **vector** is a line segment with direction. Used for velocity, forces etc.

Example: Given $O = (0, 0), A = (4, 2)$ and $B = (5, 5)$. Draw the vectors $\overrightarrow{OA}$ and $\overrightarrow{AB}$. Then write them in component notation.



$\overrightarrow{OA} = [4, 2]$

$\overrightarrow{AB} = [1, 3]$ Think B−A

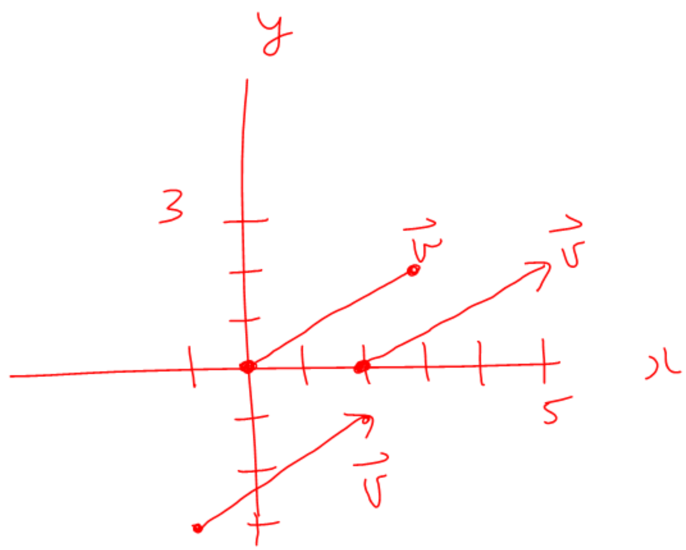
"Components of $\overrightarrow{AB}$"

Can also write $\overrightarrow{OA} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\overrightarrow{AB} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Example: Given $C = (-1, -3)$ and $D = (2, -1)$. Find $\vec{v} = \overrightarrow{CD}$ and draw it.

$$\begin{aligned}\vec{v} &= \overrightarrow{CD} \\ &= [2-(-1), -1-(-3)] \\ &= [3, 2]\end{aligned}$$

Think D-C



Infinitely-many possible answers.

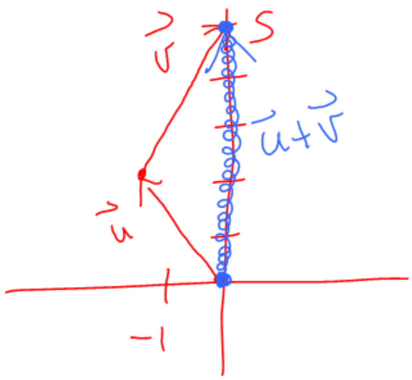
Fact: A given vector can be drawn from any initial position. Rephrased: vectors with the same length and the same direction are considered to be the same vector.

Definition: A vector is in **standard position** if it starts at the origin.

Notation: We use square brackets for vectors and round brackets for points.

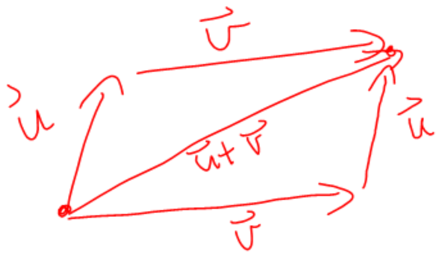
Example: Let $\vec{u} = [-1, 2]$ and $\vec{v} = [1, 3]$. Find $\vec{u} + \vec{v}$ both algebraically and geometrically.

$$\vec{u} + \vec{v} = [-1+1, 2+3]$$
$$= [0, 5]$$

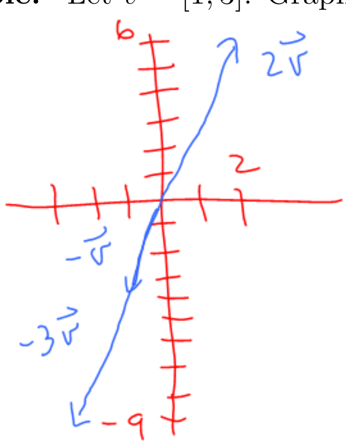


head to tail

Example: Graph $\vec{u}, \vec{v}$ and $\vec{u} + \vec{v}$ without a coordinate system.



Example: Let $\vec{v} = [1, 3]$. Graph $2\vec{v}, -\vec{v}$ and $-3\vec{v}$.



$$2\vec{v} = [2, 6]$$
$$-\vec{v} = [-1, -3]$$
$$-3\vec{v} = [-3, -9]$$

Definition: The process of multiplying a vector by a real number is called **scalar multiplication**. It produces a vector that is parallel to the original vector.

Example: Calculate $[2, 6] - [3, 4]$

$= [2,6] + [-3,-4]$

$= [-1, 2]$

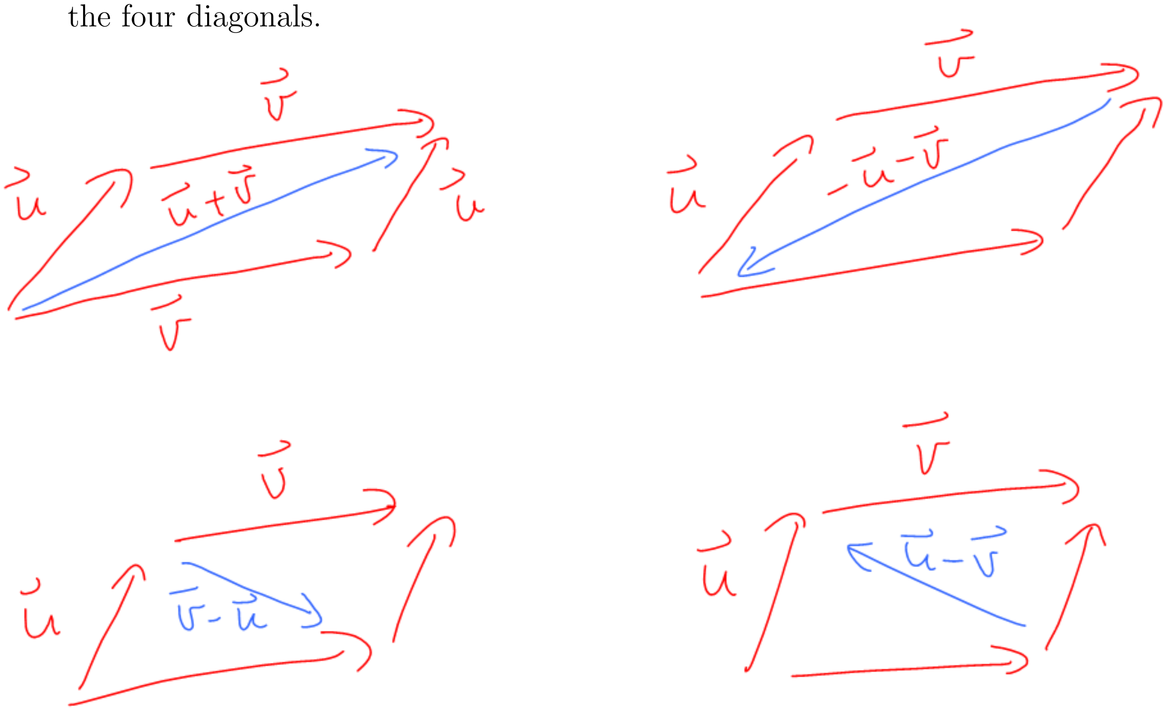
Example: Place $\vec{u}$ and $\vec{v}$ tail to tail. Find the vector that runs from the head of $\vec{v}$ to the head of $\vec{u}$.

? = backwards along $\vec{v}$
then forwards along $\vec{u}$

$= -\vec{v} + \vec{u}$

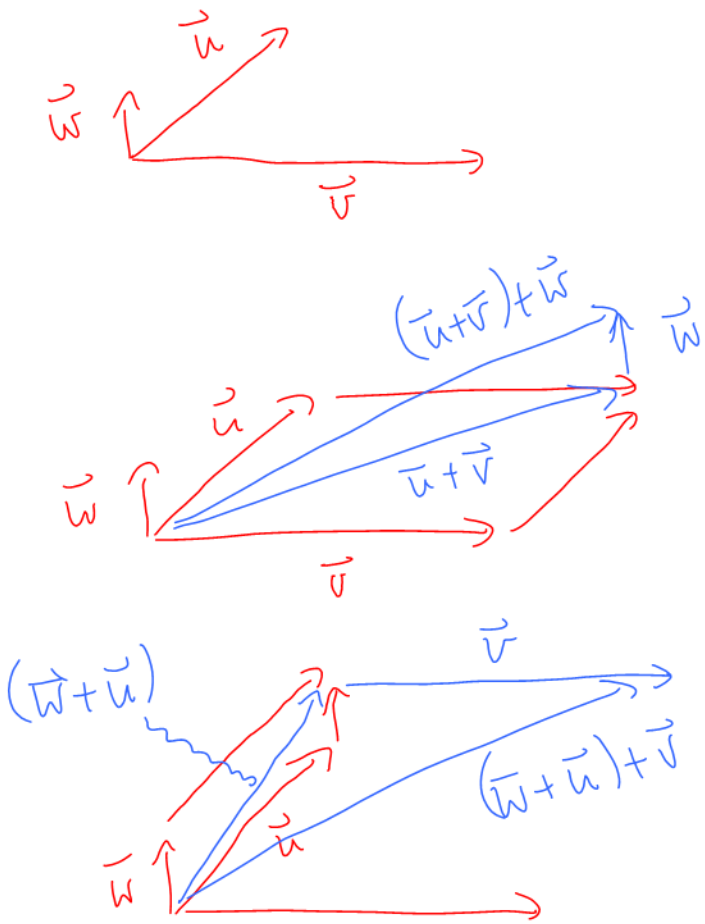
or $\vec{u} - \vec{v}$

Example: Place $\vec{u}$ and $\vec{v}$ tail to tail. Draw the parallelogram formed by $\vec{u}$ and $\vec{v}$. Label the four diagonals.



Fact: Order doesn't matter when adding vectors. For any vectors $\vec{u}, \vec{v}$ and $\vec{w}$:
$\vec{u} + \vec{v} = \vec{v} + \vec{u}$
$(\vec{u} + \vec{v}) + \vec{w} = (\vec{w} + \vec{u}) + \vec{v}$

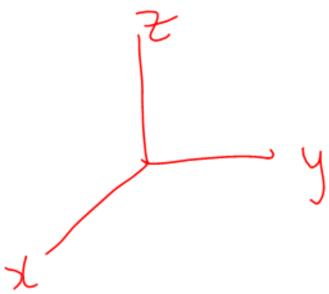
Example: Let $\vec{u}, \vec{v}$ and $\vec{w}$ be positioned tail to tail to tail. Show geometrically that
$(\vec{u} + \vec{v}) + \vec{w} = (\vec{w} + \vec{u}) + \vec{v}$

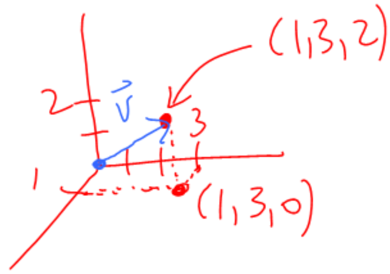


Fact: The above example illustrates that we can write $\vec{u} + \vec{v} + \vec{w}$ without any bracketing.

Definition: Consider the expression: $\vec{v}$ in $\mathbb{R}^n$. This means that $\vec{v}$ has n components, and each component is a real number.

Example: Draw $\vec{v} = [1, 3, 2]$ in $\mathbb{R}^3$.





Definition: The **zero vector** is written $\vec{0}$. Each of its components is zero. The zero vector is useful for algebra.

Example: Write the zero vector in $\mathbb{R}^2$ and $\mathbb{R}^3$.

Example: Let $\vec{u}$ be in $\mathbb{R}^2$. Show (prove) that $\vec{u} + (-\vec{u}) = \vec{0}$.