Quiz Wed Nov 8 Section 25.2

Indefinite Integral

$$\int x^2\,dx = \frac{x^3}{3} + C$$

Definite Integral

$$\int_2^3 x^2\,dx = \left[\frac{x^3}{3}\right]_2^3$$

$$= \frac{3^3}{3} - \frac{2^3}{3}$$

$$= \frac{27}{3} - \frac{8}{3}$$

$$= \frac{19}{3}$$

25.3 Area Under a Curve Cont'd

$$\int_1^5 (2x + 4x^3 + 1)\,dx = \left[x^2 + \frac{4x^4}{4} + x\right]_1^5$$

$$= \left[x^2 + x^4 + x\right]_1^5$$

$$= [5^2 + 5^4 + 5] - [3]$$

$$= 652$$

Note: Don't write $+C$ for definite integrals.

Why Not?

$$\left[x^2 + x^4 + x + C\right]_1^5$$

$$= [655 + C] - [3 + C]$$

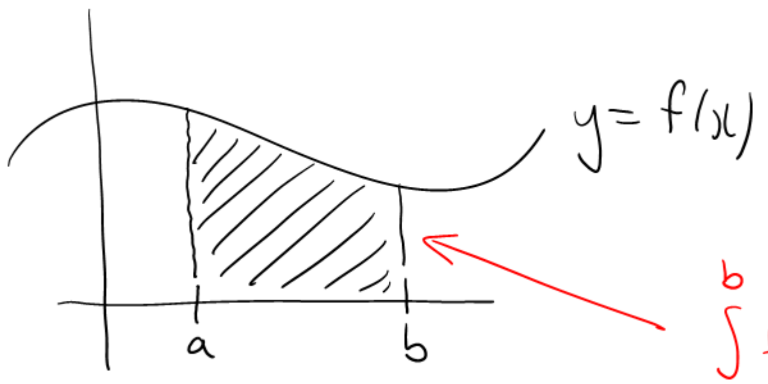
$$= 652$$

C would cancel out.

FACT

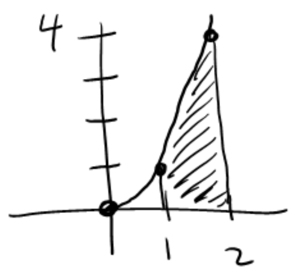
When $f(x) \geq 0$, the area under $y = f(x)$ from $x = a$ to $x = b$ is:

$$\int_a^b f(x)\,dx$$



$$\int_a^b f(x)\,dx$$

Ex: Find area under $y = x^2$ from $x = 1$ to $x = 2$.



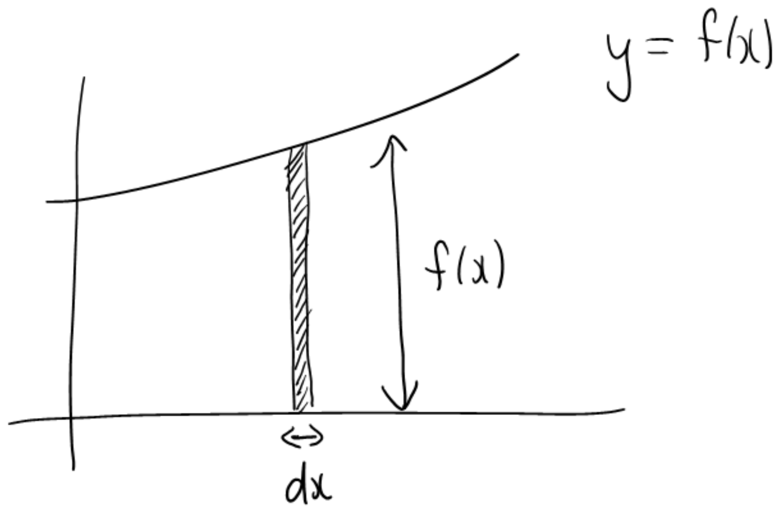
$$A = \int_1^2 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3}$$

Why does a definite integral give area under a curve?



Area of rectangle = $f(x)\,dx$

$$\int_a^b f(x)\,dx = \text{Sum of } \infty\text{-many areas of rectangles}$$
$$= \text{Area}$$

$$\int_1^2 x^2\,dx = \frac{7}{3}$$

$$\int_2^1 x^2\,dx = \left.\frac{x^3}{3}\right|_2^1 = \frac{1}{3} - \frac{8}{3} = -\frac{7}{3}$$

$$\int_1^2 (-x^2)\,dx = -\frac{x^3}{3}\Big|_1^2 = -\frac{8}{3} - \left(-\frac{1}{3}\right) = -\frac{7}{3}$$

If $f(x) \leq 0$ then the integral gives the <u>negative</u> of the area.

<u>Ex</u>: Find area under $y = \frac{1}{\sqrt{x}}$ from $x = 4$ to $x = 9$.

$$A = \int_4^9 \frac{1}{\sqrt{x}}\,dx$$

$$= \int_4^9 x^{-1/2}\,dx$$

$$= 2x^{1/2}\Big|_4^9$$

$$= 2\sqrt{9} - 2\sqrt{4}$$

$$= 2$$

25.4 Definite Integrals

Ex: Evaluate $\int_0^1 3x^2(x^3+2)^4\,dx$

$u = x^3+2$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

if $x=0$, $u=2$

$x=1$, $u=3$

$$= \int_2^3 u^4\,du$$

$$= \left[\frac{u^5}{5}\right]_2^3$$

$$= \frac{3^5}{5} - \frac{2^5}{5}$$

$$= \frac{211}{5}$$