25.2 Indefinite Integrals Cont'd

Ex: $\int x^3 (2x^4+3)^4 \, dx$

$u = 2x^4+3$

$\frac{du}{dx} = 8x^3$

$du = 8x^3 \, dx$

$\frac{du}{8} = x^3 \, dx$

$= \frac{1}{8}\int u^4 \, du$

$= \frac{1}{8}\left(\frac{u^5}{5}\right) + C$

$= \frac{(2x^4+3)^5}{40} + C$

Ex: Evaluate $\int (2+3x^2)^2 \, dx$

Expand.

$= \int (4 + 12x^2 + 9x^4) \, dx$

$= 4x + \frac{12x^3}{3} + \frac{9x^5}{5} + C$ ✓

$= 4x + 4x^3 + \frac{9x^5}{5} + C$ ✓

Ex: Evaluate $\int \frac{x^3 + x^2}{\sqrt{x}} \, dx$

Simplify.

$= \int x^{-1/2} (x^3 + x^2) \, dx$

$$= \int \left(x^{5/2} + x^{3/2}\right) dx$$

$$= \frac{2}{7}x^{7/2} + \frac{2}{5}x^{5/2} + C$$

Ex: Find y given $\frac{dy}{dx} = 7x^2$ and $(1,3)$ is on the curve.

$$y = \int 7x^2 \, dx$$

$$y = \frac{7x^3}{3} + C$$

Sub $x=1$, $y=3$:

$$3 = \frac{7}{3} + C$$

$$\frac{9}{3} - \frac{7}{3} = C$$

$$C = \frac{2}{3}$$

$$y = \frac{7x^3}{3} + \frac{2}{3}$$

Ex: $f''(x) = 3x+1$

$f(0) = 2$

$f(1) = 4$

Find $f(x)$.

$$f'(x) = \int (3x+1) \, dx$$

$$f'(x) = \frac{3x^2}{2} + x + C$$

$$f(x) = \int \left(\frac{3x^2}{2} + x + C\right) dx$$

$$\boxed{f(x) = \frac{x^3}{2} + \frac{x^2}{2} + Cx + C_1}$$

new name

$x=0$
$f(x)=2$:

$$2 = C_1$$

$$\boxed{f(x) = \frac{x^3}{2} + \frac{x^2}{2} + Cx + 2}$$

$x=1$
$f(x)=4$:

$$4 = \frac{1}{2} + \frac{1}{2} + C + 2$$

$$C = 1$$

$$\boxed{f(x) = \frac{x^3}{2} + \frac{x^2}{2} + x + 2}$$

25.3 Area Under a Curve

Definite Integral $\int_a^b f(x)\,dx$

Formula: $\int_a^b f(x)\,dx = F(b) - F(a)$

where $F(x)$ is an antiderivative for $f(x)$

$$\int_2^3 x^3\,dx = \left[\frac{x^4}{4}\right]_2^3$$

DEFINITE INTEGRAL

$$= \frac{81}{4} - \frac{16}{4}$$

$$= \frac{65}{4}$$

$$\int x^3 \, dx = \frac{x^4}{4} + C$$

INDEFINITE INTEGRAL